CONTENTS FOR APRIL, 1949

PAPERS

PAGE

Deflections of Plywood Beams Due to Moisture Content Change.
By W. E. Wilson and Laurence G. Olson 429

Modern Concepts Applied to Concrete Aggregate.
By R. F. Blanks 441

Control of the Hydraulic Jump by Sills.
By John W. Forster and Raymond A. Skrinde 469

DISCUSSIONS

The Relationship Between Pile Formulas and Load Tests.
By Robert D. Chellis 485

A Simplified Method of Analyzing Suspension Bridges.
By Hui-Sien Liu 489

Industrial Wastes: A Symposium.
By A. M. Quattlebaum 495

The Significance of Pore Pressure in Hydraulic Structures.
By L. F. Harza 497

Expansion Joint Practice in Highway Construction.
By William A. Conwell 518

The Relation of Urban Expressways to Mass Transportation Facilities.
By Elmer F. Steigelman, and Harold M. Lewis 519

Mineral Land Surveys.
By Charles P. Eager, and Robert Lenon 523

Integrating the Equation of Nonuniform Flow.
By Philip Z. Kirpich, and Paul V. Hodges 527

Cities Cannot Live Without Trucks.
By Harold M. Lewis 531

Nomographic Analysis of Rectangular Sections of Reinforced Concrete.
By Maurice Barron, and M. Hirschthal 535

Highway Bridge Floors: A Symposium.
By Louis Balog 538

CONTENTS FOR APRIL, 1949 (*Continued*)

PAGE

The Logical System of Freight Rates.
By Alexander Markowitz 545

Cadastral Surveys and Resurveys.
By Scott P. Stewart and Clarence S. Jarvis, and Roger E. Amidon 548

GUIDEPOST FOR TECHNICAL READERS
APRIL, 1949, *PROCEEDINGS*

Items of value or significance to readers in various fields are here listed. For convenience the distribution is given in terms of the Society's Technical Divisions

Technical Division	*Pages*
Air Transport	485, 545
City Planning	495, 519, 523, 531, 545, 548
Construction	429, 441, 469, 485, 518, 519, 538
Engineering Economics	519, 523, 538, 545
Highway	441, 485, 489, 518, 519, 531, 538, 545
Hydraulics	469, 497, 527
Irrigation	469, 497, 527
Power	469, 497, 527
Sanitary Engineering	495, 527
Soil Mechanics and Foundations	446, 485
Structural	429, 441, 469, 485, 489, 497, 518, 535, 538
Surveying and Mapping	523, 548
Waterways	545

(A constant effort is made to supply technical material to Society members, over the entire range of possible interest. In so far as your specialty may be covered inadequately in the foregoing list, this fact is a gage of the need for your help toward improvement.—Ed.)

HINTS TO AUTHORS

Those who are planning papers for submission to PROCEEDINGS *will expedite Committee action measurably by first studying the standard instructions as to style, content, and format. For reprints, address the Manager, Technical Publications, 33 W. 39th St., New York 18, N. Y.*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DEFLECTION OF PLYWOOD BEAMS DUE TO MOISTURE CONTENT CHANGE

BY W. E. WILSON,[1] ASSOC. M. ASCE, AND LAURENCE G. OLSON[2]

SYNOPSIS

This study was undertaken to determine reasons for the deformation of long floor panels of plywood box girder construction. Preliminary consideration revealed that the cause must be either temperature or moisture content changes. A theory was developed to show the relationship between deflection and change in temperature or moisture content for I-beams and T-beams. This development required knowledge of the relationship between length and moisture content of plywood panels. Experiments were conducted to establish this relationship for plywood of various thicknesses; and, to check the validity of the theory, experiments were also conducted to determine the deflection of beams as a function of moisture content. The results of these experiments indicate close agreement between theory and experiment limited, as may be expected, by the variation from piece to piece and from time to time of the physical characteristics of wood.

As a result of the analysis and experimental work it is concluded that, because plywood shows marked dimensional change with change in moisture content, its use in the construction of long box girders or T-beams requires the exercise of precautions to secure constant moisture content. It is further concluded that if proper moisture control is not feasible the length-depth ratios of the beams and girders must be made small enough to insure that, should moisture content change occur, the resulting change in the shape of the beam or girder would not be objectionable.

The analysis and experimental work indicate that it is feasible to calculate in advance the magnitude of the deflection to be expected in a girder or beam for a given change in moisture content. It is recommended that further experi-

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **September 1, 1949.**

[1] Pres., State School of Mines and Technology, Rapid City, S. Dak.

[2] Asst. Research Engr., Sundstrand Machine Tool Co., Rockford, Ill.

mental work be conducted to make possible the complete determination of the physical characteristics of the plywood in so far as the relationship between moisture content and dimensions is concerned.

Introduction

The construction experience which led to this study of the effect of moisture content changes in plywood beams on the deflection of that type of member is based on fabrication of plywood box girders, by two different methods, for prefabricated houses. Panels of one type, which are used in walls, are 8 ft long, 3 in. deep, and approximately 3 ft wide. These are glued by high frequency electric heating and contain nails for tacking purposes only. The second type of girder, which is 6 in. deep, 3 ft wide, and 23 ft long, is used for both floor and roof construction and is manufactured by gluing and nailing the plywood to the webs. Experience with the second type of girder led directly to the analysis and experimental work described herein.

These box girders, consisting of $\frac{1}{2}$-in. top skin, four $\frac{3}{4}$-in. by $5\frac{1}{4}$-in. web members, and $\frac{1}{4}$-in. bottom skin, had been constructed and were found to undergo a warping action sufficient to produce as much as a 1-in. to $1\frac{1}{2}$-in. deflection from their original straight form. Deflections of this magnitude were found in storage, and after the panels were installed in houses there occurred deflection changes that produced highly undesirable results including breaking of the asphalt tile which covered them.

This investigation had as its primary purpose the determination of the cause of the distortion of the plywood box girders previously described. To attain this objective it was necessary to carry out several studies having the following individual objectives:

(1) To determine the relationship between the change in the length of a piece of plywood and the change in its moisture content;

(2) To determine the theoretical relationship between the deflection of a beam or girder and the moisture content of its plywood flanges;

(3) To determine experimentally the relationship between deflection of a beam or girder and the moisture content of its plywood flanges;

(4) To compare theoretical results from the second study with experimental results from the third study, noting any discrepancies;

(5) To determine limitations on the size of plywood box girders imposed by the effect of moisture content on the beams or girders; and

(6) To determine if the deformations due to moisture content change might be controlled or if the changes in moisture content might be controlled.

Theory

To maintain as great simplicity as possible in developing the theory of the T-beam and the I-beam, which consist of solid timber webs and plywood flanges, certain simplifying assumptions are made. Because of algebraic complications, the I-beam theory yields much more complex equations than does the T-beam

theory. It was deemed advisable, therefore, further to simplify the I-beam theory. As a result of this situation, the T-beam theory is developed much more fully than is the I-beam theory, and the experimental check on the T-beam theory is more complete.

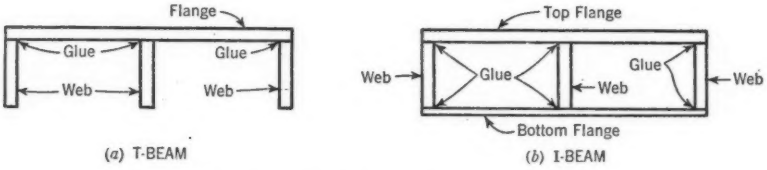


FIG. 1.—TYPES OF PLYWOOD BEAMS INVESTIGATED

For purposes of this paper a T-beam is considered to be a structural member consisting of solid timber webs glued to a plywood flange as indicated in Fig. 1(*a*). An I-beam differs from the T-beam in that it has both a top and a bottom flange (Fig. 1(*b*)).

T-*Beams.*—In developing the theory of the deformation of a T-beam due to change in moisture content the notation of Fig. 2 is employed. The following assumptions are made:

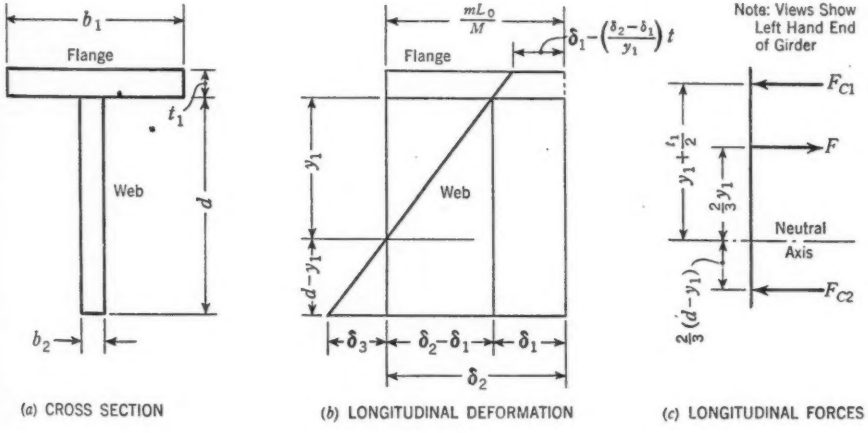


FIG. 2.—T-BEAM DIAGRAMS

1. The longitudinal deformation of a piece of plywood due to moisture content change is given by the expression:

$$L = L_0\left(1 - \frac{m}{M}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which L is the length of the plywood at moisture content corresponding to m; L_0 is the length of the plywood at zero moisture content; m is the moisture content expressed as the ratio of the weight of moisture to the oven dry weight

of the plywood; and M is a dimensionless coefficient experimentally determined as explained subsequently;

2. In bending, plane sections remain plane;
3. The deflection of the beam is small compared to its length; and
4. The plywood and the lumber are elastic, obeying Hooke's law.

Considering a beam which has been deformed, as shown in Fig. 2, because of an increase in moisture content of the flange, the first assumption and the geometry of the bent beam yield the relationship:

$$\frac{m L_0}{M} = \delta_2 \quad \ldots\ldots (2)$$

in which δ_2 represents the total elongation of the unrestrained flange due to moisture content. The second assumption yields the equation:

$$\frac{\delta_2 - \delta_1}{y_1} = \frac{\delta_3}{d - y_1} \quad \ldots\ldots (3)$$

in which δ_1 is the shortening of the flange due to the compression induced by the resisting action of the adjacent dry lumber; δ_3 is the elongation of the extreme fibers of the web; and y_1 is defined in Fig. 2(*b*). The fourth assumption yields the expressions:

$$F_{c1} = \left[\delta_1 - \frac{(\delta_2 - \delta_1)\, t_1}{2\, y_1} \right] \frac{E\, b\, t_1}{L_0} \quad \ldots\ldots (4)$$

$$F_{c2} = \frac{\delta_3}{L_0} \frac{E\, b_2}{2} (d - y_1) \quad \ldots\ldots (5)$$

and

$$F_T = \frac{\delta_2 - \delta_1}{L_0} \frac{E\, b_2}{2} y_1 \quad \ldots\ldots (6)$$

in which F_{c1} is the compressive force in the flange; F_{c2} is the compressive force in the web; F_T is the tensile force in the web; and E is the modulus of elasticity.

The conditions of static equilibrium require that

$$F_{c1} + F_{c2} - F_T = 0 \quad \ldots\ldots (7)$$

and

$$F_{c1} \left(y_1 + \frac{t_1}{2} \right) - \frac{2\, F_T\, y_1}{3} - \frac{2\, F_{c2}}{3} (d - y_1) = 0 \quad \ldots\ldots (8)$$

Eqs. 2 to 8, inclusive, are sufficient for the determination of the relationship between the seven dependent variables δ_1, δ_2, δ_3, y_1, F_{c1}, F_{c2}, and F_T in terms of the eight independent variables E, t_1, b_1, b_2, d, m, M, and L_0.

The geometry of the deflected beam (Fig. 3) yields the following expression for the maximum deflection D in terms of the elongation of the fibers and the length of beam and the location of the neutral axis:

$$D = \frac{d_2\, L_0}{8\, y_1} \quad \ldots\ldots (9)$$

Solution of Eqs. 2 to 9, inclusive, yields

$$D = \frac{m\,L^2_0}{8\,M\,d}\left[\frac{3\,\frac{t_1}{d}\left(1+2\,\frac{t_1}{d}\right)}{6\left(\frac{t_1}{d}\right)^3+9\left(\frac{t_1}{d}\right)^2+4\,\frac{t_1}{d}+\frac{b_2}{b_1}}\right] \quad \ldots\ldots\ldots (10a)$$

or

$$\frac{8\,M\,d}{L^2_0}\,\frac{D}{m} = \frac{3\,\frac{t_1}{d}\left(1+2\,\frac{t_1}{d}\right)}{6\left(\frac{t_1}{d}\right)^3+9\left(\frac{t_1}{d}\right)^2+4\,\frac{t_1}{d}+\frac{b_2}{b_1}} \quad \ldots\ldots\ldots (10b)$$

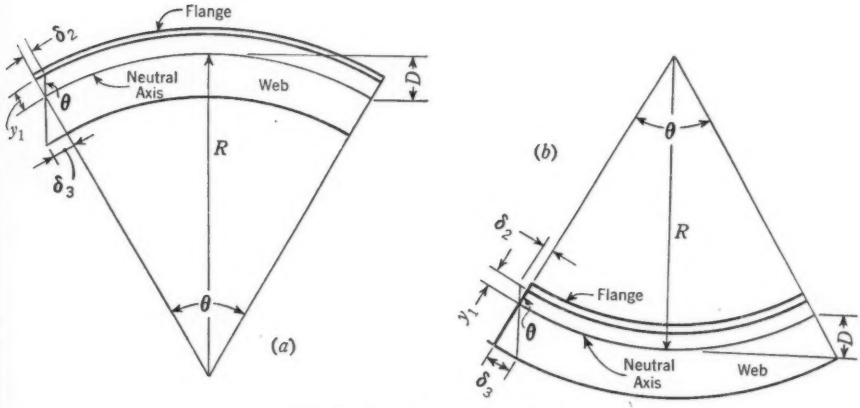


FIG. 3.—DEFLECTION OF T-BEAM

—which is represented graphically in Fig. 4, where the left-hand side appears as the ordinate; and the ratio t_1/d, as the abscissa.

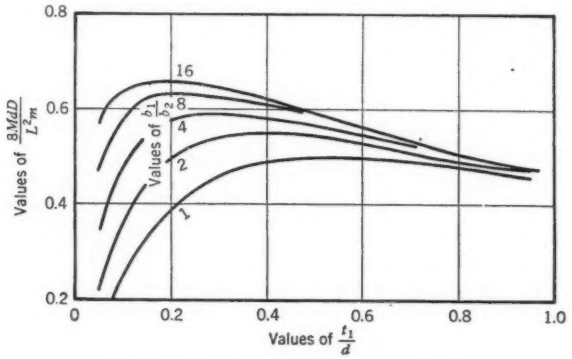
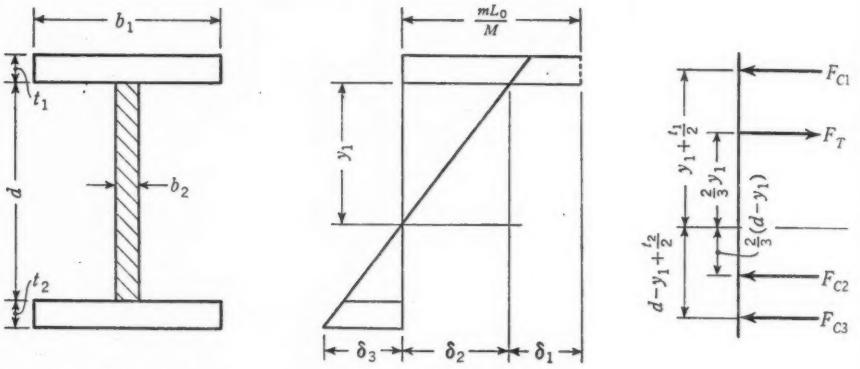


FIG. 4.—THEORETICAL DEFLECTION OF T-BEAMS

I-*Beams*.—The analysis of the mechanics of the deflection of **I**-beams is similar to that for **T**-beams and starts from the diagrammatic representation of a beam of this type (Fig. 5). The assumptions made in the analysis for

an I-beam differ from those for a T-beam only in one simplifying point related to the second assumption. It is assumed that plane sections between flanges remain plane, but that the deformation of the flanges is constant throughout the flange depth, which is small compared to the beam depth.



(*a*) CROSS SECTION (*b*) LONGITUDINAL DEFORMATION (*c*) LONGITUDINAL FORCES

FIG. 5.—I-BEAM DIAGRAMS

As a consequence of the assumptions made and the conditions of equilibrium, Eqs. 2 and 3 are directly applicable to the I-beam, as are Eqs. 5 and 6.

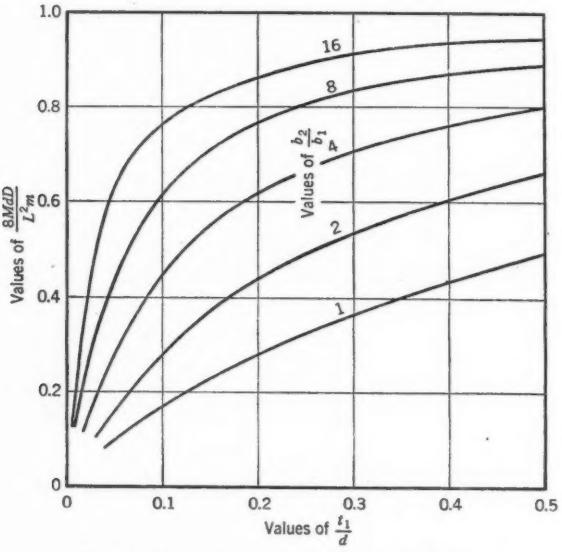


FIG. 6.—THEORETICAL DEFLECTION OF I-BEAMS

In addition, Eq. 4 is replaced by

$$F_{c1} = \frac{\delta_1 E b_1 t_1}{L_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

and an equation is required to describe the compressive force in the lower flange:

$$F_{c3} = \frac{\delta_3}{L_0} E\, b_1\, t_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Also the conditions of equilibrium require that

$$F_{c1} + F_{c2} + F_{c3} - F_t = 0 \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

and

$$F_{c1}\, y_1 - 2\, F_t \frac{y_1}{3} - \tfrac{2}{3} F_{c2}\,(d - y_1) - F_{c3}\,(d - y_1) = 0 \ldots\ldots (14)$$

Solution of Eqs. 2, 3, 5, 6, 11, 12, 13, and 14, in conjunction with Eq. 9, yields

$$\frac{m\, L^2_0}{8\, M\, d} \frac{7}{6\left(1 + \frac{5}{12}\frac{b_2\, d}{b_1\, t_1}\right)} > D > \frac{m\, L^2_0}{8\, M\, d} \frac{1}{1 + \frac{b_2\, d}{2\, b_1\, t_1}} \ldots\ldots\ldots (15)$$

Because of the complexities of the algebraic relationships it is not feasible to obtain an exact expression for D which is workable. In Fig. 6, Eq. 15 is illustrated graphically to show the minimum value of D.

Experimentation

The experimental work consisted of three parts—namely:

1. Determination of the relationship between moisture content and the length of strips of plywood which were approximately 3 in. wide, 8 ft long, and of various thicknesses;
2. Determination of the deflection of I-beams as the moisture content of one or both flanges changed; and
3. Determination of the deflection of T-beams as the moisture content of the flange changed.

The techniques employed in these programs and the data obtained are presented herewith.

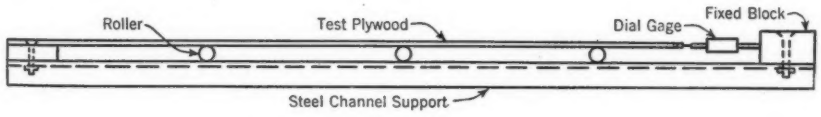


Fig. 7.—Plywood Testing Apparatus

Shrinkage with Change in Moisture Content.—The change in the length of a piece of plywood caused by changes in moisture content was determined with the equipment illustrated in Fig. 7. A strip of plywood 3 in. wide and approximately 96 in. long was bolted to a steel channel as shown. Rollers were placed under the plywood to insure freedom of movement. A block was bolted to the channel to serve as a measuring point, and a dial gage was mounted to contact this block and a steel pin which had been driven into the plywood specimen.

The plywood, which had been prepared by soaking in water for a period of several days, was mounted in the jig and a determination of the moisture content was made with an electrical resistance type moisture meter. At regular periods, measurements of moisture content at from ten to twelve points in the plywood specimen and of the shortening or the lengthening of the piece were made.

Three thicknesses of plywood were used in the study—$\frac{1}{4}$ in., $\frac{3}{8}$ in., and $\frac{1}{2}$ in. The $\frac{1}{4}$-in. and $\frac{3}{8}$-in. thicknesses were of exterior grade; the $\frac{1}{2}$-in. thickness was moisture resistant. No data were taken on other sizes since the sizes mentioned were used in the construction under study at the time of the experimentation.

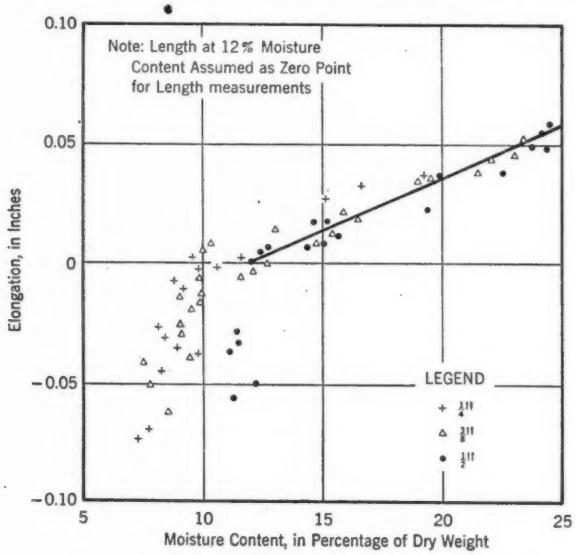


FIG. 8.—EXPERIMENTAL PLYWOOD ELONGATION DATA FOR VARIOUS THICKNESSES

The results of these measurements sre shown graphically in Fig. 8, in which the length of the plywood specimen at 12% moisture content has been arbitrarily selected as the zero point for the length measurements of the specimens. It was found that the moisture content of solid timber or plywood was rarely less than 6% or greater than 25%.

Deflection of **I**-*Beams and* **T**-*Beams.*—The test beams used in the beam deflection experiments were constructed as follows: The **I**-beams had dry (6% to 8% moisture content) 1-in. by 6-in. solid timber webs, with flanges of plywood, $\frac{1}{4}$ in., $\frac{3}{8}$ in., and $\frac{1}{2}$ in. thick, in 12-in. widths. The plywood flanges varied in moisture content, which covered two general ranges—the dry range (6% to 10% moisture content), and the wet range (20% to 25% moisture content). Six combinations of the dry webs and the wet and the dry flanges of different thicknesses were used. These beams were cold glued and nailed at 6-in. centers throughout the entire 8-ft length of the beams. The **T**-beams were constructed in a manner similar to that used for the **I**-beams, except that the

depth of the web varied from 1 in. to 6 in. and the width of the flanges varied from 3 in. to 12 in. Nine combinations of the webs and the flanges were used in the T-beam construction.

The deflection of the I-beams from their original shape was determined with the apparatus illustrated in Fig. 9, the distance from the surface of the steel

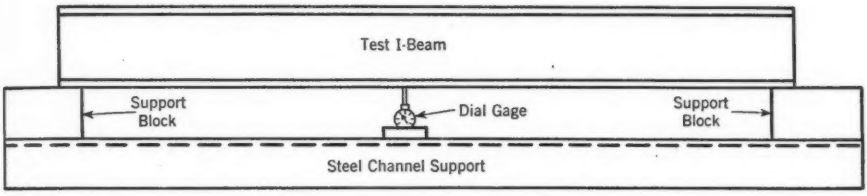


FIG. 9.—BEAM TESTING APPARATUS

channel to the bottom of the beam being measured at 12-in. intervals along the length of the beam. The shape of the steel channel surface was determined by the dial gage and the surface of a body of water at rest. The curve of the bottom of the I-beam or the T-beam was determined by calculating the distance

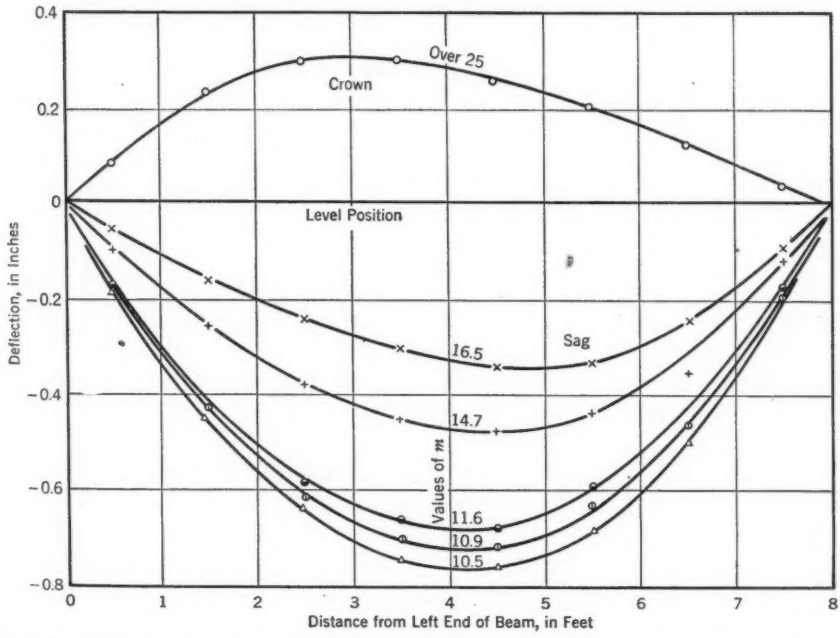


FIG. 10.—DEFLECTION CURVES FOR TEST BEAM 4 UNDER VARIOUS MOISTURE CONTENT CONDITIONS

from the bottom of the beam to a plane surface by reference to the measured distances to the steel channel surface.

A typical series of deflection curves for a beam is shown in Fig. 10. The maximum deflection D, scaled from curves such as these, is shown graphically

for I-beams and T-beams in Fig. 11 as a function of the moisture content of the test flanges, the physical dimensions of which are given in Table 1. It will

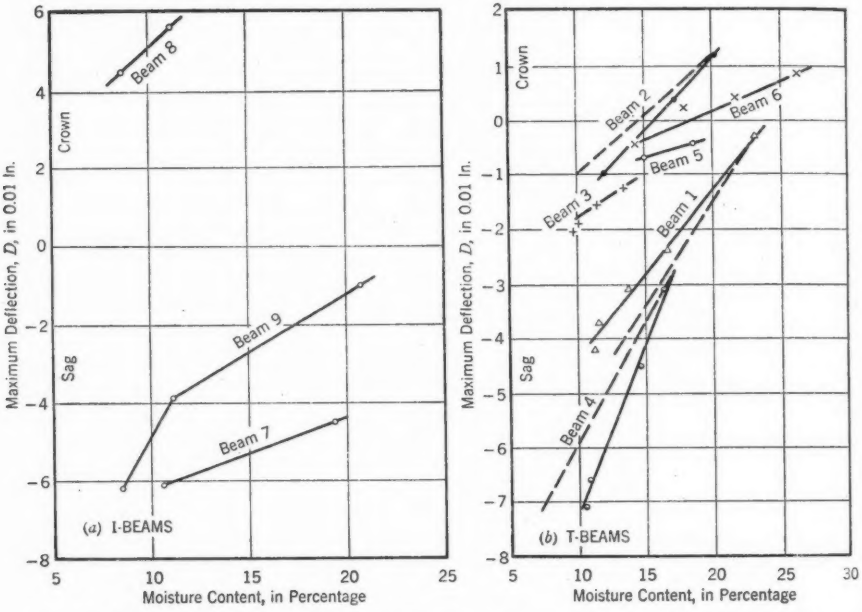


FIG. 11.—MAXIMUM DEFLECTION OF TEST BEAMS UNDER VARIOUS MOISTURE CONTENT CONDITIONS (Solid Lines, "Experimental"; Broken Lines, "Theoretical")

be noted that all beams were 8 ft long, a convenient length, since plywood is readily available in this length.

The variation of moisture content of both I-beam and T-beam flanges was produced and measured exactly as described previously for the plywood test

TABLE 1.—DIMENSIONS OF TEST BEAMS, IN INCHES[a]

Specimen No. (1)	Type (2)	b_1 (3)	b_2 (4)	t_1 (5)	t_2 (6)	d (7)
1	T	12			..	
2	T	12			..	1½
3	T	12			..	3½
4	T	12			..	
5	T	3			..	3½
6	T	6			..	3½
7	I	12				5½
8	I	12				5½
9	I	12				5½

[a] All beams were 96 in. long.

specimens using one flange of an I-beam and the only flange of a T-beam as the test flange where the moisture content varies. The moisture content of the beams decreased at such a rate that successful measurements on the specimens

easily could be obtained by making observations at intervals of from 12 hours to 24 hours.

Analysis of Data

To compare experimental results with theoretical predictions, it was necessay to determine the slope of the curves shown in Figs. 10 and 11(*a*) and the value of the coefficient M. Reference to Eqs. 11 and 15 indicates that the slope of a curve showing deflection as a function of flange moisture content should be a constant for any given beam, being a function of t_1/d, b_2/b_1, M, and d only.

Besides its dimensions, the value of the coefficient M is the only physical characteristic of the plywood that enters into consideration. The value of M is the slope of the curve shown in Fig. 8 multiplied by the length of the specimen, or the ratio of the elongation per unit change in moisture content to specimen length. Since the curve is not a single straight line, M is not a constant. However, in the range above a moisture content of 12% a straight line fits the experimental points reasonably well. The value of M corresponding to the solid straight line shown in this region is 2.25.

Using this value of M and the physical dimensions of the beams, the theoretical deflection D has been computed from Eqs. 10 and 15. The data shown in Fig. 11(*a*) represent I-beams whose deflection was due to moisture changes in one flange only. Although it was true that the moisture content of the other flanges varied a small amount the variation was so small that it might be neglected. The measured and calculated values are compared in Fig. 11(*b*). One set of points represents the calculated deflection with the full thickness of the plywood flange as the dimension t_1; the other set uses the effective thickness for the value of t_1—the effective thickness being the total thickness of the veneer, the grain of which is parallel to the longitudinal axis of the beam.

It will be noted that the experimental points are located close to the theoretical line, whereas strict agreement between theory and experiment would require that the two lines coincide. It is possible that the explanation lies in a value of M different from that determined from the experimental results shown in Fig. 8.

With the exception of the points representing specimen No. 1, close agreement between theory and experiment is indicated. In most cases the greater value of the measured deflection might well indicate a smaller value of M for the plywood used in the beams as compared with that used in the shrinkage experiments.

Conclusions

On the basis of the experimental and analytical work previously described, the following conclusions appear to be justified:

1. Although plywood in general shows dimensional stability, the elongation and the shrinkage which accompany moisture content change are sufficient in magnitude to cause significant deflections in both I-beams and T-beams.

2. The effect of moisture change can be minimized by designing short, deep beams rather than long, shallow ones.

3. Control to prevent changes in the moisture content of the flanges of a beam does not appear to be feasible.

4. Design of girders and beams of plywood box girder type must take into consideration the maximum probable change in moisture content of the plywood and the permissible deflection of the beams.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

MODERN CONCEPTS APPLIED TO CONCRETE AGGREGATE

BY R. F. BLANKS,[1] M. ASCE

SYNOPSIS

Aggregates influence the durability of concrete through numerous physical and chemical properties, most important of which are the size, continuity, and abundance of internal voids; the bonding characteristics, and the alkali reactivity. In some instances, other properties, particularly swelling characteristics associated with water absorption, may cause distress or actual deterioration of concrete.

Testing methods, as currently applied, yield statistical but not specific correlations with serviceability of aggregates; they do not reveal the fundamental qualities on which acceptance or rejection of individual aggregates should depend. The three most important properties are not determined even qualitatively. The Bureau of Reclamation (USBR) requires petrographic examination of all aggregates to be used in construction on its projects, particularly to detect potential alkali reactivity or the need for supplementary testing of materials of marginal quality.

There is great need for research to develop new testing techniques, so that reorientation of acceptance specifications can be consummated. Research on the problem of alkali reactivity, by USBR, has developed a chemical test to detect potential reactivity of aggregates; and the mechanism of this type of deterioration has been proved. Studies of capillary phenomena in rocks and concrete have revealed a diagnostic relation between pore characteristics and freezing and thawing durability. Other properties of aggregates, particularly bonding characteristics and volume change with wetting and drying, should be investigated fully.

Improvements in testing of aggregates must accompany improvements in the quality of cement, and in the techniques of mixing, placing, and curing the concrete. Only in this way will the full potentialities of concrete be realized.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **September 1, 1949.**

[1] Chf., Div. of Research and Geology, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

Introduction

The serviceability of concrete depends on many factors, among which are the quality of the aggregate and cement and the methods of fabrication, placement, and curing. With respect to durability, present standard tests fail to evaluate the quality of aggregate properly. Inadequacy of currently applied methods of selecting materials is demonstrated by repeated cases of poor service of concrete resulting from some characteristic of the aggregate; also, although less commonly, sound aggregates are rejected as a result of accepted tests, thus entailing unnecessary expense for additional exploration or the use of more costly materials.

The primary difficulty with present aggregate testing lies in the concepts that strong and durable rocks necessarily contribute to the strength and the durability of concrete, and that weak rocks contribute directly to unsoundness of concrete. Standard tests are designed to determine the quality of aggregate as rock—not as a component of concrete—whereas experience shows that good rocks do not always produce superior concrete, and weak rocks do not always cause deterioration.

Revised concepts of aggregate testing by the USBR regard the aggregates as components of concrete. Testing techniques must be designed that will determine adequately the effect of an aggregate on the properties of concrete, if satisfactory concrete is to be produced consistently.

Effect of Rock Properties on Concrete

The properties of aggregates include (*a*) those of the individual particles, and (*b*) those characterizing the entire assemblage. The latter comprise gradation, moisture content, and bulk weight. They are easily determined and interpreted and will not be considered here. The properties of aggregates related to the individual particles, their significance, and methods of measuring them will be discussed.

The individual rock particles affect quality of concrete through either physical or chemical processes. The physical properties of aggregates are related particularly to the mechanical and structural characteristics of the aggregate particles, and to a lesser extent to unusual characteristics of component minerals —such as ability to swell and to contract with wetting and drying. Chemical properties relate more specifically to the stability of the constituent minerals and rock substances while they are enclosed by Portland cement. Consequently, the chemical and physical properties of aggregate particles depend on petrographic characteristics. However, rocks of a given type do not always possess the same properties, for their properties can be changed by weathering or solution by rain or ground water, or by fracturing and jointing related to the geologic history of the region. Thus, basalts can be hard or soft; and rhyolites can be deleteriously reactive with high-alkali cements, or they may be innocuous if the reactive glass has been crystallized into stable minerals by metamorphic processes. Therefore, the quality of aggregates must be evaluated by measurement of certain chemical and physical properties which are significant to concrete.

Size, Abundance, and Continuity of Pores.—The voids or spaces within solid materials exist in several forms. They may be isolated more or less by intervening impermeable material as, for example, gas bubbles (vesicles) in glassy lavas like obsidian. They may be ramifying passageways, or approximately plane fissures, or fractures. The voids in rocks may be empty or partly or completely filled with chemically precipitated minerals such as quartz, or insoluble residues such as clay.

The size, abundance, and continuity of the voids influence the physical and chemical stability of the rock; they are the most important physical properties of aggregate particles. These characteristics of pores affect the strength of any material; they control water absorption and permeability; and they determine the quality of bond with cement. Consequently, they control durability under freezing and thawing conditions and strongly influence the rate of chemical alteration.

The durability of solid materials during freezing and thawing is controlled by the freedom with which they are permeated by water. Obviously, materials which are impermeable are not disrupted by freezing. However, highly porous, absorptive rocks likewise will not be disrupted if freezing is progressive and proceeds slowly enough to permit escape of water in excess of that necessary to fill the void spaces. Susceptible to breakdown with freezing and thawing are rocks having those intermediate degrees of permeability which allow water to enter the material but not to drain freely. Evidence in the literature (1)[2] and experimentation by the USBR indicate that, under natural conditions of freezing, voids less than 0.005 mm in diameter, and particularly those less than 0.004 mm in diameter, will drain effectively only at hydrostatic pressures that exceed the tensile strengths of some rocks and concrete.

Important to the durability of concrete is the relative size of continuous voids in the aggregate particles and of those in the hydrated cement paste. Unless the concrete remains under hydrostatic head, the water content of the aggregate will be controlled by this relative pore size. Large voids in the aggregate will be drained of water, even if initially filled, through progressive withdrawal by capillarity into the hydrating cement paste. Conversely, voids in the aggregate smaller than those in the cement paste will remain filled or will tend to become filled by extraction of water from the capillaries of the cement paste. Of course, the relative size of pores in the cement paste might vary considerably with continued hydration or alteration and dissolution.

This interchange of water between aggregate particles and cement paste may affect, significantly, the freezing and thawing durability of concrete, because the saturated particles will be more susceptible to breakdown than would unsaturated ones. Furthermore, large voids in unsaturated aggregates will accommodate the water migrating through the concrete during progressive freezing, whereas voids filled with water or voids of very small size or limited continuity might block the penetrable capillaries through which water is advancing during freezing, consequently causing rupture at or near the aggregate-to-paste contact (2).

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

The relation of size, abundance, and continuity of voids to bond of the aggregate particle with cement will be discussed subsequently.

Bonding Character.—The tenacity of the bond between aggregate and cement paste is extremely important to the quality of concrete. The properties of this aggregate-cement bond depend on surface texture and chemical stability of the aggregate. Chemical reactions that dissolve or decompose (weaken) the aggregate obviously reduce strength of bond, but reactions that form new compounds of considerable strength and elasticity may weld the aggregate to cement. Reactions of the former type are exemplified particularly by the reaction between aggregate and cement alkalies. Although they undoubtedly exist, reactions of the latter type have not been identified.

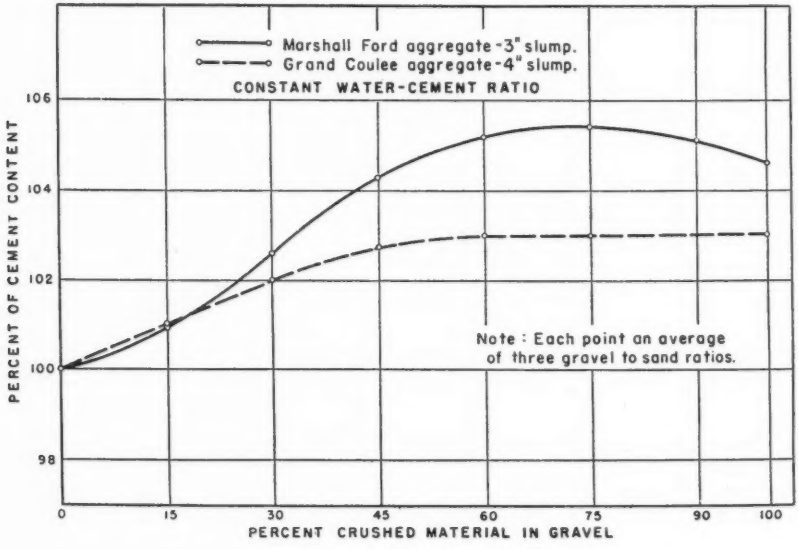


FIG. 1

Two aspects of surface texture exert an influence on bond—the surface roughness and the pore characteristics of the zone underlying the surface of the particle. Salients and depressions on the particles, particularly when the sides of these roughnesses are almost perpendicular to the general surface, assist the adherence of the paste to the aggregate. Undulatory roughness is less helpful and may even be harmful to bond as the mortar changes in volume. Rugosity is probably less significant than physical penetration of cement into the aggregate.

Particle Shape.—Flat or elongated particles in aggregate tend to decrease workability and thus to require the use of more sand, cement, and water (3). Moreover, they tend to reduce bulk weight and decrease compressive strength. During the placement of the concrete, if flat particles become oriented with their large faces in a horizontal position, bled water will be entrapped under them, a condition which prevents the development of good bond.

Sharp, angular rock fragments may require increased mortar to produce adequate workability and thus may significantly increase cement and water content (Figs. 1 and 2) if a constant water-cement ratio is to be maintained.

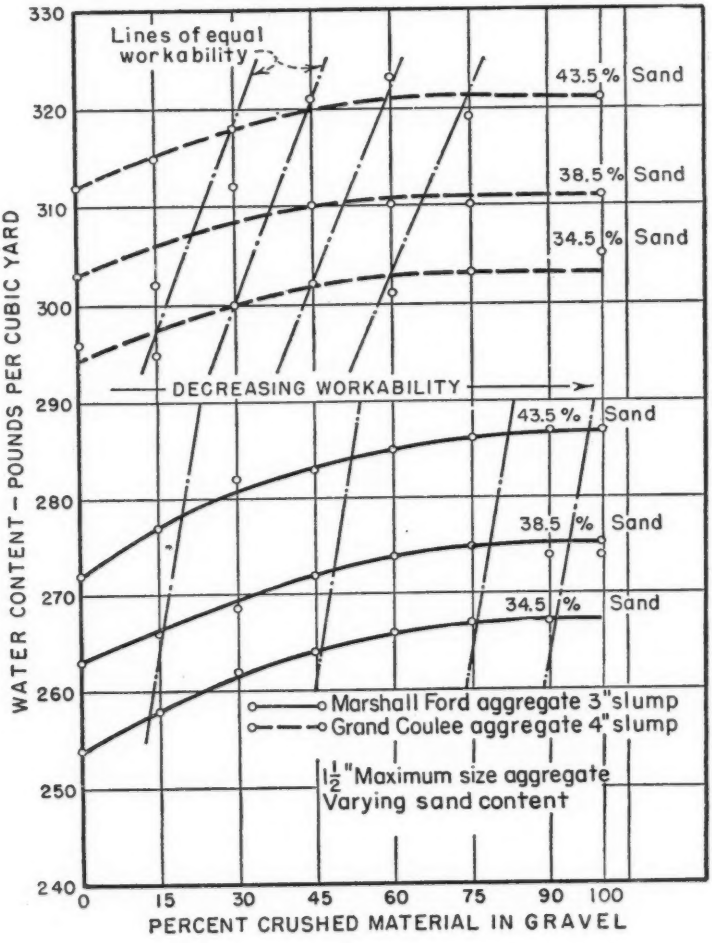


FIG. 2

Thermal Expansivity.—Thermal expansivity of aggregates affect concrete in two ways:

1. Through the influence of the average coefficient of expansion on the expansivity of the concrete as a whole; and
2. Through the action of individual particles whose coefficients differ widely from the coefficient of the cement paste.

Rarely can the effect of the expansivity of individual components of the aggregate on the concrete be measured. Failure of concrete has been attributed to thermal properties of the aggregate (4); but, in general, the effects of ex-

pansivity are masked because they merely augment, significantly or insignificantly, other deteriorative actions. It is becoming continually more evident that failure of concrete commonly results from a combination of processes which, acting individually in comparable degree, would be ineffectual.

Aggregate particles with unusually low expansivities are more objectionable than are those with unusually high expansivities, since few rocks and minerals possess higher coefficients than hydrated Portland cement. Most destructive to concrete are the tensile strains produced in the mortar when the temperature is declining, possibly being accompanied by development of tensile fractures in the cement paste.

The surprisingly rapid deterioration associated with the use of the so-called "sand-gravel" aggregate from the Republican River in Kansas (5) appears to result from a combination of several characteristics of the aggregate. The aggregates are only slightly or moderately reactive with high-alkali cements, but they contain high proportions of individual crystals of orthoclase and microcline feldspars whose coefficient of thermal expansion is very low (0.0×10^{-6} to 0.6×10^{-6} per degree Fahrenheit) (6)(7) in one crystallographic direction. These minerals constitute high proportions of the coarse sand, ranging from 34% to 45% of the No. 4 to No. 8 fraction in the samples studied (Table 1).

TABLE 1.—PETROGRAPHIC ANALYSIS OF SAND, REPUBLICAN RIVER, CLOUD COUNTY, KANSAS (PERCENTAGES BY WEIGHT, NO. 4 TO NO. 8 FRACTION)

Item	Rock types	No. 6609A	No. 6609B	No. 6609C
1	Orthoclase and microcline	45.0	38.0	34.2
2	Granites	8.0	5.7	6.9
3	Quartz	45.0	52.5	55.6
4	Basic volcanic rocks	0.5	1.6	0.8
5	Siliceous limestones	0.5	....	0.8
6	Chalcedony	1.0	2.2	1.7

In such proportion, the great difference in thermal coefficients of particle and cement will create considerable stress in the cement paste with decline in temperature.

Among common rocks, the thermal coefficient of expansion ranges from about 0.5×10^{-6} to 9.0×10^{-6} per degree Fahrenheit (8). Rocks that commonly possess exceptionally low coefficients (1.0×10^{-6} or less) include granite, limestone, and marble. The thermal coefficient of hydrated neat Portland cement usually is between 5.9×10^{-6} and 9.0×10^{-6} per degree Fahrenheit (8). The freezing and thawing durability of concrete is not recognizably affected by moderate variations in expansivity of the aggregates in the presence of other influencing factors (Table 2).

Properties Affecting Volume Change.—Two sets of physical properties adversely influence volume change of concrete during wetting and drying—those which increase expansion, and those which increase or decrease shrinkage. Concrete may be distressed by local expansions caused by excessive expansion of aggregate particles as they become wet; with drying, these same particles will contribute to excessive shrinkage. In addition, particles that are especially

compressible increase drying shrinkage, because they fail to restrain the normal drying shrinkage of the cement paste (9).

Clay minerals of the montmorillonite type (bentonitic) expand and contract significantly with wetting and drying. The expansion is accompanied by potential pressures which occasionally are several thousand pounds per square inch (10). An altered dolerite, apparently sound according to standard tests, was found to contain about 30% of interstitial clay (the mineral nontronite).

TABLE 2.—EFFECTS OF THE THERMAL EXPANSIVITY OF AGGREGATE ON THE DURABILITY OF CONCRETE MAY BE MASKED BY OTHER FACTORS

Concrete	Type of aggregate (crushed) in the concrete	Thermal coefficient of the aggregate,[a] α	Cycles of Freezing and Thawing of the Concrete	
			To 25% loss by weight	To 0.3% linear expansion
A	Diorite No. 1	4.7	13	6
	Quartz diorite	4.4	25	19
	Limestone A	2.3	44	41
	Diorite No. 2	3.8	59	45
	Limestone B	1.8	84	88
B	Limestone C	4.4	18	10
	Graywacke	5.1	105	65
	Dolomite	4.5	535	540
C	Limestone C	4.4	40	50
	Graywacke	5.1	148	134
	Dolomite	4.5	351	351
D	Quartz	5.7	40	31

[a] According to the relation $\alpha = \frac{\Delta L}{\Delta T} \times \frac{10^6}{L}$, in which L is the length of the rock specimen, ΔL is the change in length, and ΔT is the change in thermometer temperature, expressed in degrees Fahrenheit. The expectable coefficient of expansion for hydrated, neat Portland cement ranges from 5.9×10^{-6} to 9.0×10^{-6} per degree Fahrenheit (8).

Tests indicated that the rock expanded and contracted 0.06% linearly with wetting and drying, and after repeated cycles an increment of length remained. Concrete beams containing the dolerite as aggregate lost more than 68% of their original dynamic modulus of elasticity in twenty cycles of freezing and thawing. Some limestones, although sound according to standard tests, contribute to deterioration of concrete when their interstitial clay swells as it absorbs water from the concrete. Some sandstones expand as much as 0.08% linearly with wetting (11), probably because of interstitial clay.

Strength and Elasticity.—Most rocks possess strengths and elasticities considerably greater than those of concrete. The range in crushing strength of rocks generally acceptable as concrete aggregate is from 10,500 lb per sq in. to 28,700 lb per sq in. (6); yet in individual cases, these rocks are incapable of producing concrete with adequate strength and durability.

Conversely, weakness of rocks may not be reflected proportionately in the strength of the concrete. For example, a very soft, porous, pumice that could be easily crushed in the fingers and which included numerous particles that were so porous as to float on water possessed a "structural strength" in mortar equivalent to approximately 78% that of the sand used in construction of Grand

Coulee Dam in Washington, one of the best aggregates tested in the USBR laboratories. After 28 days, the average compressive strength of standard 2-in. by 4-in. mortar cylinders containing fine pumice was 3,680 lb per sq in.—equal to that obtained on mortars containing some natural sands.

Other Properties.—Specific gravity of aggregates has a marked effect on the unit weight of concrete and for that reason specific gravity of aggregates sometimes is of importance in the design of gravity structures or of structural members that should be unusually low in weight. The specific gravity of aggregates must be known for purposes of mix design, but the estimation of the soundness of aggregates from specific gravity is a tenuous extrapolation. It is true that apparent specific gravity will decrease as porosity increases, but porosity can be determined from this measurement only if the absolute specific gravity is known also. Even if the absolute and apparent specific gravities are known, only porosity, but not the size, number, or continuity of pores, will be calculable. Experience indicates that tests of specific rock types or aggregates of similar lithology will yield a correlation between specific gravity and durability; but the same correlation cannot be carried over to aggregates of other lithology. Particles with very low specific gravity commonly move to the surface and thus cause poor finish and reduced abrasion resistance.

Specific heat and heat conductivity of aggregates strongly affect the rate of temperature change in concrete and the rate at which heat is transmitted through concrete (12). These factors are important in the preparation of design and specifications for massive structures, but are of limited significance for small structural members. To the extent that the temperature of concrete is changed more rapidly or by a greater amount, then the thermal expansivities of the aggregates exert a stronger influence on the concrete.

The properties of aggregates are changed by the presence of coating materials. These materials commonly change adversely the physical or chemical properties of the aggregate, but the change is caused by the same properties that affect quality of aggregate particles as a whole. In the experience of the Denver (Colo.) laboratories of the USBR, coatings are composed of silt, clay, gypsum, impure carbonates of lime and magnesia, iron oxides, opal, manganiferous substances, soluble phosphates, or mixtures of these substances.

Chemical Properties of Aggregates.—The traditional view regarded aggregates as inert fillers which passively contributed weight and bulk to the concrete. Actually, few, if any, aggregates are inert while enclosed by cement. Any type of chemical reactivity will cause a series of complex, mutually dependent changes in the concrete, but four general types of chemical activity appear to be identifiable:

First, the aggregates may contain substances that are soluble in the aqueous solution permeating the concrete. Leaching of these substances will increase porosity of the concrete and decrease its strength. The dissolved substances can form efflorescence on and near the surface of the concrete, and thus cause scaling.

Second, soluble substances or products of decomposition of aggregate particles may modify normal hydration of the cement. Thus, organic substances

Fig. 3

may react with solutions in the concrete, and consequently cause development of abnormal hydration products. Strength and durability are reduced by such changes, and failure of the concrete can result. Oxidation of pyrite or marcasite (FeS_2) in aggregate causes local disintegration, pop outs, and staining of concrete; this action is related to production of sulfuric acid (H_2SO_4), which decomposes and dissolves adjacent cement paste, and hydrated iron oxides (which may occupy greater volume than the original crystals of the sulfide). Sulfates may decompose slowly and dissolve after incorporation in the concrete, causing progressive expansion of the concrete, through the formation of calcium sulfoaluminate by reaction with hydrated aluminates of the cement (Fig. 3).

TABLE 3.—LINEAR EXPANSION (%) OF MORTARS CONTAINING ZEOLITES OR ZEOLITIC ROCK WITH HIGH-ALKALI OR LOW-ALKALI CEMENT[a]

Replacement in crushed quartz (%)	CEMENT 2742; 0.12% K_2O AND 1.30% Na_2O				CEMENT 2735; 0.14% K_2O AND 0.04% Na_2O			
	Age, 1 month	Age, 3 months	Age, 6 months	Age, 1 year	Age, 1 month	Age, 3 months	Age, 6 months	Age, 1 year
(a) ANALCITE; GOLDEN, COLO.								
5	0.004	0.015	0.017	0.033	−0.002	0.001	−0.002	0.007
10	0.003	0.014	0.015	0.031	−0.005	0.000	−0.003	0.006
20	0.002	0.014	0.016	0.031	−0.004	0.000	−0.002	0.005
50	0.001	0.010	0.013	0.028	−0.001	0.001	−0.003	0.009
75	0.002	0.007	0.008	0.022	−0.006	−0.001	−0.003	0.010
100	0.000	0.005	0.003	0.017	−0.004	0.000	−0.002	0.010
(b) STILBITE; NOVA SCOTIA								
5	0.006	0.014	0.020	0.028	−0.001	0.003	0.003	0.004
10	0.007	0.014	0.019	0.026	−0.003	0.001	0.002	0.004
20	0.006	0.016	0.016	0.019	−0.001	0.002	0.003	0.006
50	0.005	0.013	0.013	0.017	....	....	....	....
75	0.001	0.005	0.006	0.009	....	....	....	....
100	0.000	0.003	0.004	0.008	−0.002	0.007	0.002	0.004
(c) WEATHERED ANORTHOSITE; SOLEDAD CANYON, CALIFORNIA								
5	0.007	0.009	0.015	0.027	−0.003	−0.007	−0.007	−0.004
10	0.006	0.007	0.013	0.029	−0.002	−0.005	−0.006	−0.001
20	0.003	−0.002	−0.001	0.009	−0.003	−0.007	−0.011	−0.004
50	−0.006	−0.013	−0.014	−0.006	−0.004	−0.011	−0.014	−0.007
75	−0.013	−0.019	−0.023	−0.015	−0.006	−0.013	−0.017	−0.009
100	−0.020	−0.028	−0.030	−0.026	−0.007	−0.015	−0.020	−0.007

[a] Bars, 1 in. by 1 in. by 10 in., with indicated materials replacing crushed quartz in all size fractions. Concrete mix, 1:2, cured 7 days in fog room at 70° F; then stored at 100° F in sealed cans, in the presence of excess water. Aggregate graded 20% by weight each of sizes Nos. 8, 16, 30, 50, and 100, no pan.

Third, certain rocks and minerals are attacked by alkalies released during hydration of high-alkali cements (13)(14)(15). Physical-chemical phenomena initiated by the reactions lead to expansion, cracking, and decreased elasticity and strength of the concrete. These phenomena will be discussed in detail subsequently.

Fourth, concrete may be affected by base-exchange phenomena involving components of the aggregate and the caustic solution within the cement paste. This process usually involves exchange of alkalies in the aggregate for the

calcium contained in the solution. The released alkalies attack aggregate particles susceptible to cement-aggregate reaction, or they may form deposits wherever the water is evaporated from the concrete. A somewhat altered granitic rock (granodiorite) used as aggregate in construction of a thin dam in southern California apparently has participated in base-exchange reactions of this kind, because efflorescent deposits of impure alkali carbonates occur on the downstream face of the structure and scaling of the concrete has taken place. A zeolite similar to laumontite (or leonhardite) (Ca, Na_2) $Al_2Si_4O_{12} \cdot 4H_2O$, occurring in the rock called anorthosite is associated with failure of cast stone and stucco in southern California. G. F. Loughlin (16) attributed the failure to volume change of the zeolite with change in water content. It is possible, however, that base-exchange reactions contributed to the deterioration. Chemical studies of the zeolite containing anorthosite indicate that the rock has a significant proportion of replaceable bases (1.4% by weight), mainly calcium, sodium, and magnesium.

Although zeolites may contribute to deterioration of concrete through base exchange, they do not cause significant expansion of moist-stored mortars when present alone or mixed with nondeleterious aggregate material (Table 3). In theory at least, zeolites with exchangeable sodium or potassium could contribute to alkali-aggregate reaction in concrete or mortar containing reactive aggregate by release of the alkalies upon adsorption of calcium released by hydration of the cement. However, contractions as great as 0.030% were measured after 6 months on the bars containing the weathered anorthosite (with included laumontite) possessing significant base-exchange capacity.

Tests of Concrete Aggregate for Quality

For many years, engineering laboratories have applied a series of standardized tests to determine the quality of aggregates proposed for use in construction. The tests usually applied include the determination of: (*a*) Specific gravity and absorption; (*b*) soundness, usually with sodium or magnesium sulfate; (*c*) abrasion resistance; (*d*) organic impurities in sand; (*e*) quantities of coal or lignite, clay lumps, and shale; and (*f*) properties of standard mortars to find the structural strength of fine aggregate.

The results of these tests correlate statistically with the service of concrete containing the aggregates, but cannot be depended on to diagnose the quality of a particular aggregate. For this reason, engineers have come to rely on tests and examination of concrete wherever possible. If an aggregate has been used in construction, service histories will be available; or, if laboratory facilities are at hand, the aggregates may be incorporated in concrete or mortar specimens. Service histories of concrete containing the proposed aggregate and subject to conditions similar to those anticipated at the project under investigation are most diagnostic. On the other hand, service histories can be dangerously misleading if data regarding the cement, mix design, or placement and curing methods are incomplete, or if the conditions of service are not similar.

Tests of concrete or mortar containing proposed aggregates will indicate the quality of the material if the mixes are prepared and testing procedures are

designed to simulate anticipated conditions on the job. Influence of the aggregate on durability, strength, elasticity, permeability, expansivity, thermal properties, drying shrinkage, or bulk weight can be determined in the laboratory. However, the time necessary for adequate tests of concrete frequently precludes application of such methods to studies of concrete aggregate. During a 6-month period in 1946, the USBR laboratories in Denver were called on to test more than 200 different concrete aggregates. Neither personnel nor facilities are available in any laboratory for tests of concrete on a comparable scale. Consequently, for most purposes, the concrete engineer must select aggregate by methods other than those which involve testing of concrete.

The standard tests usually applied to unconfined aggregates do not measure directly the properties of aggregates that affect the quality of concrete. Properties as fundamental as size, continuity, and abundance of pores; bonding characteristics; and thermal expansivity are not measured even qualitatively.

Specific Gravity and Absorption Determinations.—Specific gravity and absorption commonly correlate with freezing and thawing durability if lithologically similar materials are tested, but no fundamental relation to durability can be adduced. Pumice commonly used as lightweight aggregate might have a bulk specific gravity (American Society for Testing Materials (ASTM) designation: E12) of 1.0 and a 24-hour absorption of 24%, and yet produce a concrete which is sound in freezing and thawing. Determinations of specific gravity and absorption by standard methods (ASTM designations: C127–42 and C128–42) do not distinguish those aggregates whose specific gravity is low and whose absorption is high because of the presence of a few unsound particles, from those which contain an abundance of particles whose specific gravity is but slightly low and absorption but slightly high. For this reason, absorption and specific gravity of lithologically complex aggregates cannot be related quantitatively to soundness. Statistical correlations indicate that rigorous application of specified limits for specific gravity and absorption does not assure elimination of unsound aggregates and may cause rejection of some satisfactory aggregates.

Accelerated Soundness Tests.—Accelerated soundness tests involving use of sodium or magnesium sulfate (ASTM designation: C88–41T) are commonly used to estimate susceptibility of the aggregate to breakdown under freezing and thawing conditions because the sulfate soundness tests are more rapid and involve less equipment than do actual freezing and thawing of aggregates. Breakdown is believed to result from hydration and consequent expansion of crystals of Na_2SO_4 or $MgSO_4 \cdot H_2O$, following reimmersion of the samples after the drying period (17). This mechanism is certainly different from the processes by which freezing and thawing of water cause breakdown. Consequently, the results of the sulfate soundness tests do not parallel the results of freezing and thawing soundness tests. Moreover, as pointed out previously, the durability of aggregate does not always correlate with the durability of concrete. In addition to these fundamental deficiencies of the test, extraneous variations may vitiate results. Inconsistencies in the sulfate soundness tests arise from: (*a*) Differences in the types of containers used; (*b*) differences in techniques of drainage following the immersion period; (*c*) inadequate temperature control; (*d*) variations in length of the drying period; (*e*) differences in techniques of

screening the samples; and (*f*) differences in grades of salt and concentration of the solutions used in the test. Frequently, aggregates that fail the test prove to be satisfactory, and those that pass actually might contribute to the breakdown of concrete during freezing and thawing.

Abrasion Test.—The Los Angeles abrasion machine (ASTM designation: C131-39) largely has replaced the Deval machine (ASTM designation: D2-33) in tests of concrete aggregate. Extensive tests have established a general relation between strength of concrete and the percentage of wear in the Los Angeles abrasion test (18). The Los Angeles abrasion test yields information of value on the degree to which aggregates containing inordinate quantities of soft particles can be processed effectively. Specifically, aggregates that lose excessively in the first one hundred revolutions, but that lose less than 40% in five hundred cycles, commonly can be made suitable for use by scrubbing or special processing.

Breakdown of the sample is produced mainly by impact of the cast-iron or steel spheres (19), and hence is not predominantly an abrasion loss. The test reveals the presence of soft, friable, or exceptionally brittle particles; but limited loss in the test does not assure high quality of the aggregate. A crushed, milky quartz used as a control aggregate shows only 33.8% wear in the five hundred revolutions of the abrasion test; yet it consistently fails to produce durable concrete (Table 2); the low "structural strength" of the highly quartzose Ottawa sand is well known.

Test of Organic Impurities in Sand.—Organic materials in aggregates are partly dissolved by the solution of sodium hydroxide used in the standard test for organic impurities in sand (ASTM designation: C40-33). However, dark coloration of the solution merely indicates the need for further testing (particularly in mortar) rather than the rejection of the aggregate. Some organic substances are innocuous, yet produce a strongly colored solution. Organic substances occur as woody particles, coatings on aggregate particles, or as waxy or petroliferous materials disseminated through rock particles.

Determination of Content of Unsound Particles.—The standard tests to determine content of coal or lignite (ASTM designation: C123-39), and shale (ASTM designation: D546-41) involve gravity separation of the sample in a heavy liquid. Presumably, the coal or lignite, and the shale, float on the surface, but other particles of low specific gravity also will separate in the same manner. Clay lumps are identified in the standard test (ASTM designation: C142-39) as those particles that can be broken up into fine material with the fingers.

These three tests have been replaced in the USBR laboratories by petrographic examination, thus eliminating erroneous classification of lightweight or soft particles.

Petrographic Examination.—For several years petrographic examinations have been made of all aggregates proposed for use in concrete construction on projects of the USBR; but, with passing time and increasing knowledge of the significance of the petrographic properties of aggregates, the results of these examinations have become more significant and increasing reliance has been placed on them. The techniques and methods of petrographic examination of

coarse aggregates have progressed from mere visual examination to detailed analysis and description of complex aggregates (8)(20).

Petrographic examination is used to supplement the results of the standard tests or to judge the need for special tests. Suspected chemical instability or physical unsoundness can be checked by tests of mortar or concrete. Detailed knowledge concerning the lithologic composition of aggregates commonly will permit precise determination of the cause for observed deficiencies, so that ameliorative measures can be undertaken. Petrographic examination is the only method heretofore available by which alkali-reactive aggregates can be identified, quickly and reliably.

Research on Concrete Aggregates

Deficiencies of the standard tests have led to attempts at development of additional methods by which the physical and chemical properties of aggregates could be determined. Because of their great significance, work was conducted first on alkali reactivity and on the characteristics of pores which occur in solid materials. These researches are being continued, but results to date (1949) will be summarized.

Research on Alkali-Aggregate Reaction.—Soon after the experiences of Thomas E. Stanton, M. ASCE, in California (13) were made available, the USBR became intensely interested in the problem of alkali-aggregate reaction because of experiences at Parker Dam on the Colorado River in California and Arizona (14)(20)(21). Adequate facilities and specialized personnel were procured in 1941 to undertake a thorough investigation of the types of aggregate material capable of participating in this reaction, the mechanism producing the expansion and decline in strength of the concrete, and possible ameliorative methods where reactive aggregates are encountered. (The problem of cement-aggregate reaction represents an interesting example of the manner in which engineering problems can be attacked by scientific methods. The research herein summarized, and reported elsewhere (22)(23) in 1948, was obtained by highly trained and specialized scientific personnel. Integration of research and coordination of science and engineering have yielded results of immediate practical value.)

It has been well established that the deterioration manifests itself as decline in strength of the concrete and as expansion. Outwardly, the concrete exhibits typical patterns of cracking, which may be accompanied by exudation of siliceous gels at the surface. These criteria furnish the engineer with methods for preliminary diagnosis of the disease, but complete diagnosis often requires thorough laboratory study. Even under favorable circumstances it is not always possible to obtain an unambiguous diagnosis if the deterioration has been extremely severe, because secondary effects may mask the primary cause.

Numerous examples of concrete structures affected by alkali-aggregate reaction are now recognized, and many of these are related to the aggregates of several specific river systems and drainage areas. The map (Fig. 4) shows only the locations of structures that are definitely known to be affected by alkali-aggregate reaction (22*a*). West of the states bordering the Mississippi River there are several areas wherein no affected structures are indicated, notably

Texas, Oklahoma, and New Mexico; most of Nevada and Utah; and Montana, North Dakota, and South Dakota. However, this condition does not necessarily imply that deleterious aggregates are absent in these areas. It merely indicates that no affected structures in these areas have come to public attention. Acid to intermediate volcanic rocks are present in some of these areas and, where significant quantities of these rocks have been used as aggregate in

FIG. 4.—LOCATION OF STRUCTURES THAT HAVE BEEN AFFECTED BY ALKALI-AGGREGATE REACTION

combination with high-alkali cements, deterioration caused by alkali-aggregate reaction is certain to occur. East of the Mississippi River, volcanic rocks are comparatively rare at the surface, but opaline or chalcedonic cherts are widespread and are known to have caused deterioration of concrete through alkali-aggregate reaction in Georgia and Alabama. Although only a few spots appear on the map, again it should be recalled that this must not be taken to imply that alkali-aggregate reaction can be disregarded in this large area.

Some of the earliest laboratory investigations conducted by the USBR were directed toward the identification of rocks and minerals that react deleteriously and their comparative reactivities. Although previous work (13)(14)(21)(24)(25) had indicated that opal, a phyllite, and certain intermediate to acidic volcanic rocks were susceptible to attack, the degree of reactivity of most rocks and minerals was unknown. Therefore, samples of many common rocks and minerals were obtained from various sources and, after crushing and grading, were fabricated into mortar bars with low-alkali and high-alkali cements. Each sample of test aggregate was examined by USBR petrographers to determine its mineralogical and textural properties.

As a result of these studies, it was demonstrated that all silicate and silica minerals react with alkalies of Portland cement; but the greater number of minerals and rocks react insignificantly, without causing deleterious expansion or other distress accompanying cement-aggregate reactions. The rocks and minerals that contribute to deterioration of concrete in this way can be detected

by petrographic examination. The reactive minerals, and their chemical and physical characteristics, are (22*b*):

Reactive minerals	Chemical composition	Physical character
Opal	$SiO_2 \cdot nH_2O$	Amorphous
Chalcedony	SiO_2	Cryptocrystalline fibrous
Tridymite (26)	SiO_2	Crystalline

Reactive rocks and their reactive components are:

Reactive rocks	Reactive component
Siliceous rocks—	
Opaline cherts	Opal
Chalcedonic cherts	Chalcedony
Siliceous limestones	Chalcedony, opal, or a combination of both
Volcanic rocks—	
Rhyolites and rhyolite tuffs	
Dacites and dacite tuffs; Andesites and andesite tuffs	Volcanic glass, devitrified glass and tridymite
Metamorphic rocks—	
Phyllites	Hydromica or very fine-grained quartz
Miscellaneous rocks—	
Any rocks containing veinlets, inclusions, or grains of the foregoing rocks or minerals.	

To be sure, aggregate material may be deleterious for other reasons. In addition to the aforementioned reactive rocks, for example, artificial glasses, such as pyrex glass, are known to be deleteriously reactive. Whereas the volcanic types listed are known to be deleteriously reactive, basalts and at least certain phonolites are known to be innocuous, and data regarding trachytes and latites are lacking.

Reliance on the technical skill and experience of petrographers to interpret the potential reactivity of aggregates has several obvious disadvantages. To overcome these difficulties, experimentation was directed toward the development of a chemical test capable of predicting the potential reactivity in quantitative terms. Several varieties of physical and chemical tests have been advocated during the past several years (24)(27)(28)(29) but none of these yields information diagnostically related to the fundamental causes of expansion, decline in strength, etc. Moreover, all are impractical for one or more reasons such as:

1. They require too much time;
2. They depend on properties that cannot be measured quantitatively; or
3. They cannot be applied simultaneously to all the different rock constituents of a complex aggregate.

As early as 1934, R. E. Stevens (30) demonstrated that many common minerals are slightly soluble in water and that slurries of finely ground minerals quickly become alkaline. Some minerals yield very mildly alkaline solutions

whereas others produce moderately strong alkalinity. Attempts to correlate these types of data on aggregates with their deleteriousness, as indicated by mortar bar expansions, produced unsatisfactory results and these experiments were abandoned. However, the studies aided in formulation of later concepts.

Basically, testing a concrete aggregate for reactivity is simple. It should consist merely of crushing the aggregate to an appropriate size, adding some specific amount of alkaline solution, and measuring the effects of the interaction. Unfortunately, early tests of this type gave results that correlated rather poorly with the service histories and mortar bar data on the aggregates. Opal will usually give a diagnostic test by almost any procedure, as it did in most leach tests (27)(28)(29), but the problem of differentiating between the less reactive and the innocuous materials was not resolved by these early methods, which were based on determination of the quantity of silica dissolved by the caustic solution. The experiments demonstrated that some chemically innocuous aggregates (such as basalt) released more silica into solution than did certain moderately deleterious aggregates.

During tests of materials in different size fractions, it was discovered that the innocuous aggregates released from two to four times more silica into a strongly alkaline solution from pulverized material than from somewhat coarser sizes. This result indicates a normal relationship between the surface area and the quantity of silica released into solution. However, release of silica did not occur in this straightforward way when deleterious aggregates were tested. In these experiments, the reduction of active alkali in the solution as well as the amounts of silica taken into solution from the aggregate were determined analytically. The measurements indicated that the potency of the caustic solution was reduced more rapidly than silica was dissolved from deleterious materials, and thus demonstrated that the earlier leach tests were not diagnostic of deleteriousness because the amount of silica held in solution is not proportional to the amount of reaction. A successful method of testing requires measurement of two factors—the effect of the solution on the aggregate and the effect of the aggregate on the solution.

Correlation of the chemical data with mortar bar expansion is presently obtained by comparing the quantity of silica released from an aggregate with the reduction in the alkalinity of a 1N sodium hydroxide solution during the reaction. For the test, specially constructed stainless steel vessels containing the aggregate material and caustic solution are immersed for 24 hours in a liquid bath controlled at 80° C ± 1°. The test requires rigorous control and precise analytical methods, to yield reliable results. Diagnostic correlations have been obtained (Fig. 5) for more than seventy rocks, minerals, sands, and gravels on which mortar bar data are also available. In Fig. 5, only a few of the tested materials are indicated merely to demonstrate the expectable range of analytical results and the separation of innocuous from deleterious aggregates.

The chemical data obtained during the development of an aggregate test for reactivity, combined with important information obtained from microscopic investigations and other experiments, form the basis for a general concept of the mechanism by which alkali-aggregate reaction causes a breakdown of concrete. The process of deterioration may be summarized as follows: During

the mixing period and the subsequent several hours, a large part of the alkalies of the Portland cement dissolve in the liquid phase. As hydration progresses, the calcium aluminates and the silicates of the paste extract water, thus causing the liquid to become even more strongly caustic. The caustic solutions then attack susceptible particles of the aggregate and produce alkalic silica gels. These gels have a high affinity for water. As they imbibe water, they swell and produce hydrostatic pressures within the confining concrete.

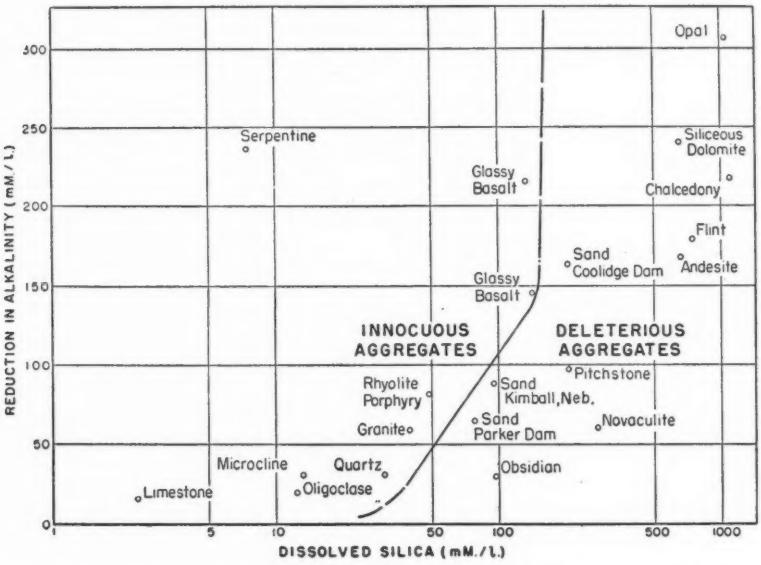


FIG. 5.—CHEMICAL INTERACTION OF CONCRETE AGGREGATE AND SODIUM HYDROXIDE (NaOH) SOLUTION

Osmotic pressures in excess of 550 lb per sq in. have been measured (22*c*) in the laboratory (Fig. 6), and similar pressures have been computed in accordance with recognized laws of physical chemistry. Thus, pressures that exceed the tensile strength of concrete can be expected from the formation and the hydration of the gel. As a consequence of rupture of the paste, microfractures develop and the pressures are momentarily relieved by the intrusion of the gels into the microfractures. These gelatinous materials may ultimately exude at the surface of moist-stored concrete affected by this malady. The microfractures account qualitatively for both the increase in volume of the concrete and the decline in strength; a quantitative demonstration of this fact would be rather difficult to obtain.

Capillarity and Its Relation to Durability of Solid Materials.—Formation of ice within solid materials results in breakdown through the operation of one or more of three separate processes:

a. The development of excessive hydrostatic pressures within materials of low permeability during progressive freezing;

b. Formation of ice lenses; or

c. The excessive thermal expansion of ice filling voids while temperatures are rising, in the range below the freezing point.

The effects of hydrostatic pressure during progressive freezing cannot be doubted, but this mechanism alone probably will not explain all the observable variation in disintegration. Stephen Taber (31) has proved the development of ice lenses in soils of low permeability. Both of these mechanisms require that pores within the materials be minute, for neither is operative if the pores

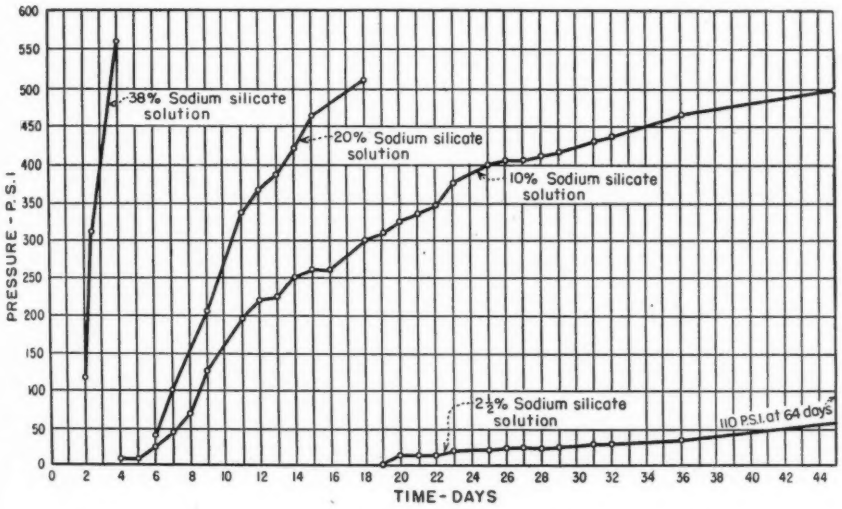


FIG. 6

are predominantly large. The effect of excessive thermal expansivity of ice crystals is probably less significant than the other two mechanisms; yet the thermal properties of ice may cause distress of rock or concrete, especially if the temperature changes below the freezing point are large. The thermal coefficients of linear expansion of rocks and hydrated Portland cement rarely exceed 9.0×10^{-6} per degree Fahrenheit (8), but in the range from 32° F to −4° F, the coefficient of linear expansion of ice is 28×10^{-6} per degree Fahrenheit (32).

The extent of breakdown in solid materials due to hydrostatic pressure, on the one hand, or to lens formation, on the other, is unknown. Breakdown of materials of low permeability through the development of high hydrostatic pressures is undoubtedly widespread, and plausibly will account for observed distress of concrete (2) and disintegration of rock in many situations (11). Mr. Taber developed the concept of lens formation specifically for soils, but A. F. Gill and J. F. J. Thomas (33) applied it to studies of masonry, bricks, and concrete. Subsequently, in 1944, A. R. Collins (34) utilized it to explain the disruption of porous concrete subjected to freezing and thawing. Mr. Taber (35) found the potentialities for stress development in natural materials to be relatively small, since he inhibited the formation of ice lenses by surface loads

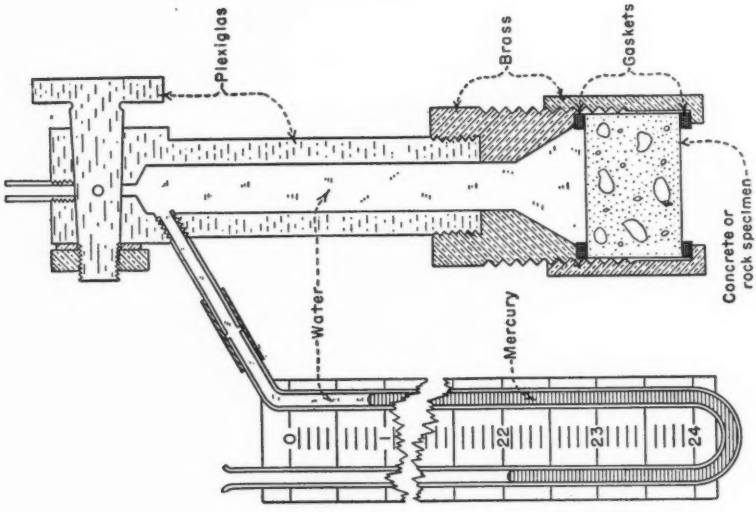

Plexiglas
Brass
Gaskets
Concrete or rock specimen
Water
Mercury
0
22
23
24

FIG. 8

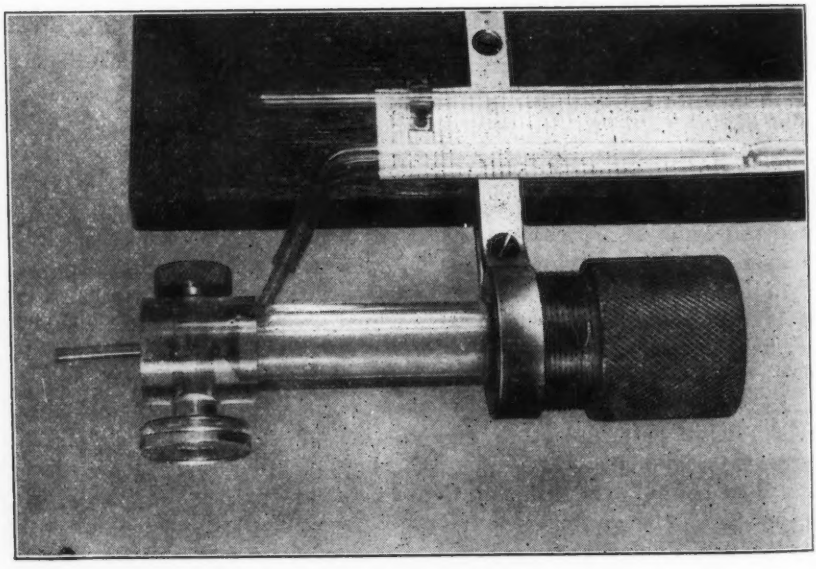


FIG. 7

equivalent to 15 kg per sq cm (213 lb per sq in.), although he believed that tensile stresses as high as 1,350 kg per sq cm (approximately 19,000 lb per sq in.) are possible in ideal systems. Nevertheless, Mr. Collins considered that the stress accompanying lens formation was the result of crystal growth and that it was limited in magnitude only by the melting pressure of ice at any given temperature (namely, 30,000 lb per sq in. at −8° F).

Experiments in the USBR laboratories, conducted under conditions that preclude the development of the high hydrostatic pressures associated with low permeability, indicate that lens formation can cause disruption of inherently weak rocks of low permeability or of rocks containing fractures. Lens formation also occurs in unset (36*a*) or very porous concrete (34), and porous bricks (33), but it would appear that this mechanism is operative only where tensile strength is small.

Because freezing and thawing durability is related to the size, continuity, and abundance of pores, an index of durability for any particular material can be obtained by measuring the rate of evaporation from saturated specimens and the force associated with capillary movement of water into the material. By modifying apparatus used in the Department of Scientific and Industrial Research, London (37), England, an instrument (Fig. 7 and 8) was designed in the USBR laboratories to measure these properties of rock, concrete, and mortar.

The apparatus is composed of a clamp in which the specimen is sealed, a water reservoir, and a mercury manometer. The specimens tested in the apparatus are cylinders 1.5 in. in diameter and up to 1 in. thick. The specimen is clamped into the apparatus and sealed by ring gaskets in such a way that air or water cannot pass around it. The cell above the specimen and all space above the mercury in the manometer are filled with distilled water. Air is eliminated from the specimen (by saturation under vacuum) and from the reservoir prior to the initiation of the test.

After the cock at the top of the reservoir has been closed, water that evaporates from the surface of the specimen will be replaced by water from the reservoir because of capillary attraction. However, the system is open to atmospheric pressure at the other side of the manometer, so that the forces exerted by the capillary openings can be measured provided they are at less than atmospheric pressure. The rate of evaporation is controlled by the relative area of continuous capillaries exposed at the lower surface. The pressure in the reservoir will continue to decline either until it equals the vapor pressure of water at the temperature of operation, or until the air is able to force the water from the largest pores and thus to prevent further increase of the pressure deficiency within the reservoir.

It is clear that materials containing abundant, continuous pores of small size will be indicated by the rapid increase and the ultimately high magnitude of pressure deficiency. Materials characterized by a few large pores will be indicated by slow increase in pressure deficiency, and this deficiency will still be comparatively small when air begins to pass through the specimen into the reservoir. All combinations of rate of rise and ultimate magnitude of pressure deficiency are possible for different materials. In every case, the ultimate pres-

sure deficiency is controlled by the minimum diameter of the largest continuous channel way in the specimen. Results of measurements on rocks and concretes are indicated in Figs. 9 and 10; supplementary data are compiled in Tables 4 and 5.

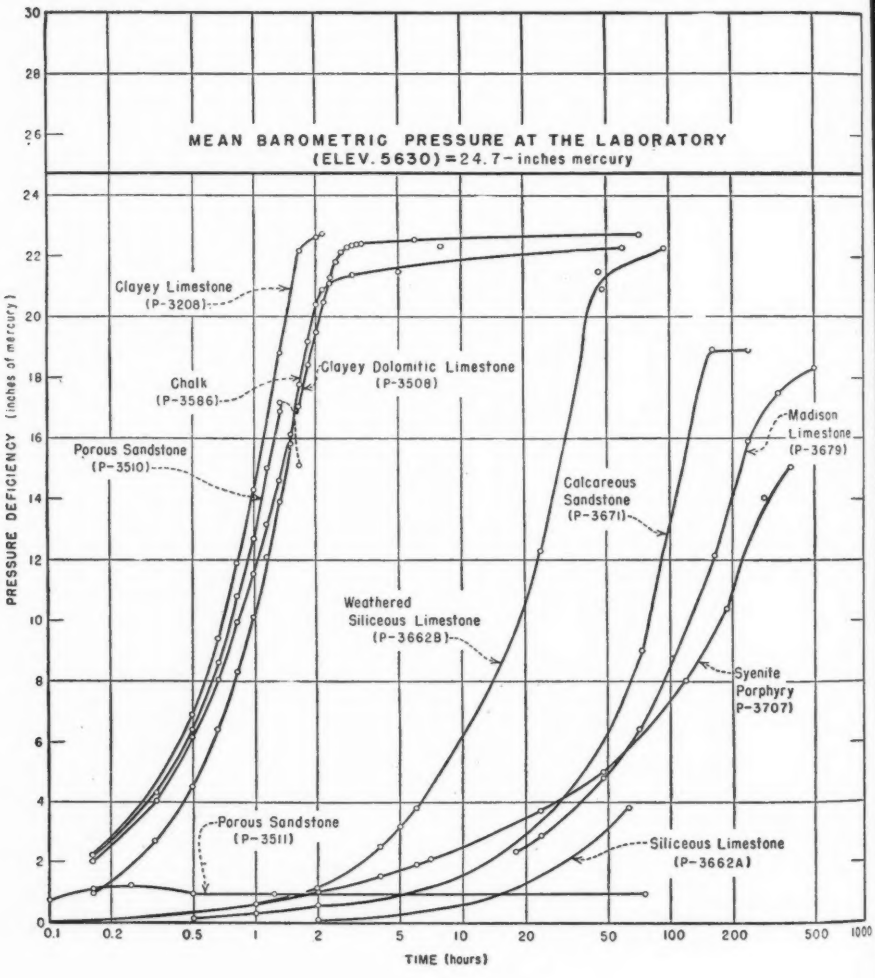


FIG. 9

The rocks that failed during the freezing and thawing test (Nos. P-3208, P-3586A, P-3508, and P-3662B, Table 4) caused rapid increases in pressure deficiency, which ultimately exceeded 22 in. of mercury. The weathered siliceous limestone (P-3362B) is represented by a curve (see Fig. 9) below and to the right of curves representing the other unsound rocks. This rock disintegrated more slowly and less completely than the other unsound rocks. Apparently, three of the sound rocks (P-3662A, P-3679, and P-3707) would

sustain high pressure deficiencies if the test were continued long enough. Evidently, their soundness in freezing and thawing is controlled largely by their inability to become critically saturated while immersed in water.

The two sandstones (P-3510 and P-3511A, Table 4) are friable and weak (having lost 95% or more in the Los Angeles abrasion test), yet they lost very

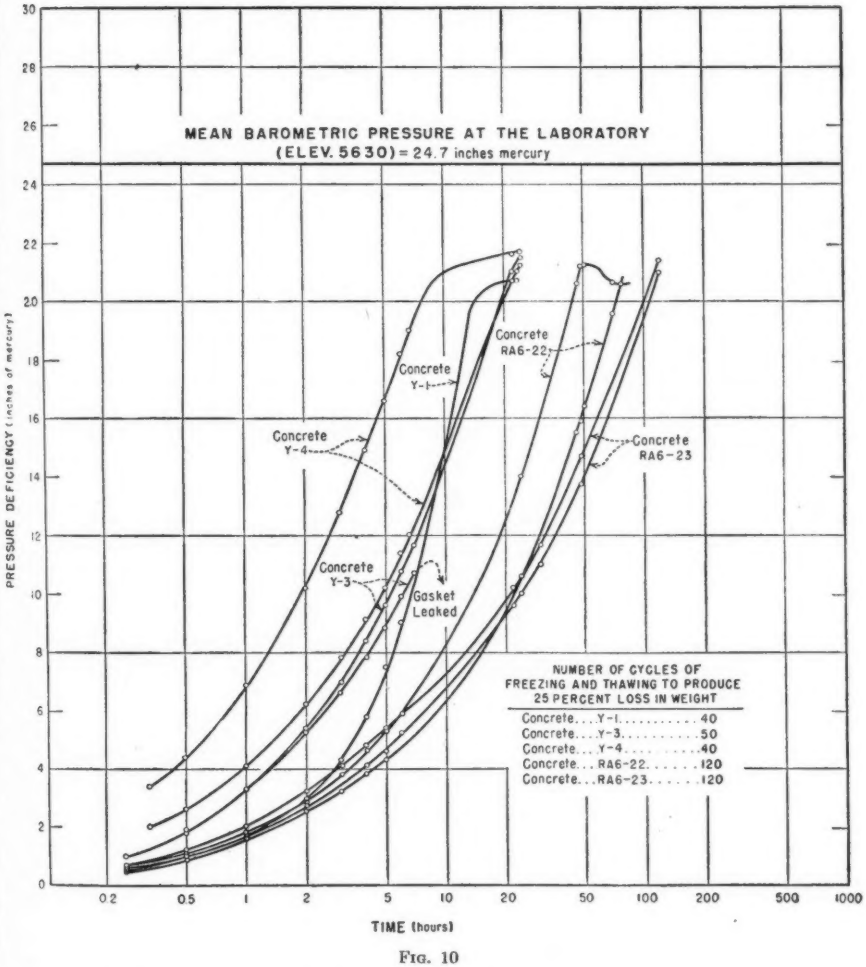


FIG. 10

small amounts of material in the freezing and thawing test. Their soundness is evidently related to the large pores which can drain effectively during progressive freezing. One of these sandstones (P-3510) sustained a pressure deficiency of 17.1 in. of mercury, before air was forced by atmospheric pressure through the specimen. The rapid rise in pressure deficiency represents the high effective porosity. The sound calcareous sandstone (P-3671) apparently is characterized by relatively few pores, yet the pores are comparatively large, the ultimate pressure deficiency being 18.9 in. of mercury.

TABLE 4.—OBSERVATIONS ON ROCKS USED IN THE MEASUREMENT OF CAPILLARITY

No.	Type	Description	%[a]	Results of soundness tests[b]
P-3208	Clayey Limestone:	Firm, fine grained, finely porous, highly argillaceous	5.2	Specimens cracked at four cycles; broke down to small pieces at six cycles
P-3586A	Chalk:	Firm, exceedingly fine grained, finely porous; contains 5% clay	16.4	Specimens broke down to small chips and sludge during four cycles
P-3508	Clayey Dolomitic Limestone:	Firm, exceedingly fine grained, finely porous; contains 13% of silt, clay, and fine sand	....	Specimens broke down to small pieces and sludge during twenty cycles
P-3662B	Weathered Siliceous Limestone:	Moderately hard, brittle, fine grained, red; opal, chalcedony, clay, and sand constitute 42% of rock	6.6	Specimen split in two, and one part broke down during fifteen cycles into several small pieces
P-3662A	Siliceous Limestone:	Moderately hard, brittle, fine grained, yellowish buff; opal, chalcedony, clay, and sand constitute 46% of rock	2.7	Specimen lost 1.4% by spalling during fifteen cycles
P-3510	Porous Sandstone:	Friable, porous, fine grained, quartzose sandstone; lost 96.7% in five hundred cycles of Los Angeles abrasion test	3.6	Specimens lost 0.1% to 0.7% by granulation during fifteen cycles
P-3511A	Porous Sandstone:	Friable, porous, medium grained, quartzose sandstone; lost 95.0% in five hundred cycles of Los Angeles abrasion test	7.6	Specimens lost 0.2% to 2.0% by granulation during fifteen cycles
P-3679	Madison Limestone:	Hard, massive, slightly porous, fine grained oolitic	0.2	Specimens lost 0.2% by spalling at edges in fifteen cycles
P-3707	Syenite Porphyry:	Hard, porous, fine grained; contains large phenocrysts (crystals)	2.0	No loss in fifteen cycles
P-3671	Calcareous Sandstone:	Hard, slightly porous, fine grained, cemented by calcium carbonate	1.7	No loss in fifteen cycles

[a] 24-hour absorption. [b] Results of freezing and thawing in 24-hour cycles; freezing in cold room at −10° F. thawing in tap water.

TABLE 5.—MIX PROPERTIES OF CONCRETES USED IN THE MEASUREMENTS OF CAPILLARITY

Concrete[a]	Aggregate[b]	Cement	Mix, in Parts by Weight[c]				Slump (inches)	Unit Weight	Water Content	Cement ($\frac{bbl}{yd^3}$)	Cycles[d] (%)
			w/c	C	S	G		Pounds per cubic yard			
Y-1	P	5784	0.76	1.00	3.85	3.85	$2\frac{1}{4}$	144.5	313	1.10	40
Y-3	L	5784	0.61	1.00	2.65	2.65	5	147.0	350	1.53	50
Y-4	L	4701	0.61	1.00	2.65	2.65	1	147.0	350	1.53	40
RA6-22	P	5784	0.55	1.00	2.65	2.65	$6\frac{1}{2}$	145.7	315	1.52	120
RA6-23	P	5784	0.55	1.00	2.65	2.65	$6\frac{1}{2}$	145.7	315	1.52	120

[a] Cylinders, 3 in. by 6 in. [b] Maximum size of aggregate, $\frac{3}{4}$ in.; Definitions: P = Platte River aggregate, Denver, Colo.; and L = crushed limestone. [c] Definitions: w/c = water-cement ratio; C = cement; S = sand; and G = gravel. [d] Cycles of freezing and thawing to 25% loss of weight. Specimens were cured 14 days in a fog room at 70° F. Then they were stored at 50% relative humidity at 70° F. The freezing and thawing soundness test was conducted after 180 days; and the capillarity experiment was run at an age of 2 years for concretes Y-1, Y-3, and Y-4 and at an age of 3 years for the remaining two concretes.

Calculations, based on the relation of surface tension of water to size of capillary and height of rise, indicate that the pressure deficiency of the three rocks (P-3208, P-3586A, and P-3508, Table 4), which sustained a pressure deficiency of 22.7 in. of mercury, is equivalent to that which could be created by a system of pores 0.0037 mm in diameter. The pressure deficiencies sustained by the sandstones (P-3671, P-3510, and P-3511A) are equivalent to pores 0.0044 mm, 0.0050 mm, and 0.0846 mm in diameter, respectively.

Concrete specimens were tested similarly in the apparatus (Fig. 10 and Table 5), and three (Y-1, Y-3, and Y-4) were found to be unsound, equivalent specimens having lost 25% by weight at forty cycles, fifty cycles, and forty cycles, respectively. The remaining two concretes (RA6-22 and RA6-23, Table 5) are satisfactory in quality, since equivalent specimens lost 25% by weight only after one hundred and twenty cycles of freezing and thawing. The rates of rise and the magnitudes of pressure deficiency are comparable to those measured during testing of the rock specimens. The pore characteristics of the rocks are therefore comparable to those of the concrete. Difficulty was experienced in testing concrete Y-1, Table 5, for in three attempts gasket leaks occurred, and reliable measurements could not be obtained for one specimen of the two prepared. The slow initial rise in pressure deficiency may be caused by the precipitation of soluble salts within the pores of the specimen during the wetting and the drying entailed by repeated testing.

As for the unsound rocks, poor durability of the concrete is indicated by rapid rise to high pressure deficiency (20.7 in. of mercury or greater). Sound concretes are indicated by much slower increases in pressure deficiency but the values ultimately attained are high (20.6 in. of mercury or greater). Some variation in results is related to position in the original concrete from which the tested specimen was cut, as is implied by the separation of curves representing specimens from different ends of the same 3-in. by 6-in. cylinder (Fig. 10). The rate and the magnitude of rise are indifferent to the kind of aggregate and cement, apparently being related only to durability.

The ultimate values of pressure deficiency (20.6 in. to 21.7 in. of mercury) measured on the concrete correspond to pores 0.0045 mm to 0.0040 mm in diameter. Thus, the controlling pores in the cement paste are larger than the controlling pores in the four unsound rocks tested. Capillary movement of water from the cement paste into the aggregate particles of weathered or argillaceous limestones is therefore possible. Consequently, these particles can attain and retain a high degree of saturation while enclosed by cement paste. The pertinence of this conclusion is indicated in the section of this paper concerning size, abundance, and continuity of pores in aggregate.

The apparatus used in these experiments can be modified so as to measure the abundance of voids of different sizes. However, techniques required for these measurements are time consuming, and will be unnecessary if experience continues to indicate that durability of rock and concrete can be predicted adequately from a function of effective porosity and pore size.

Conclusions

Many physical and chemical properties of aggregates affect quality of concrete. Most important are: (*a*) The size, continuity, and abundance of internal voids; (*b*) the bonding characteristics; and (*c*) the alkali reactivity. Other properties (especially volume change with wetting and drying) occasionally are critically important. The standard tests generally applied to determine acceptability of aggregates are inadequate. They yield data only statistically, not specifically related to the fundamental quality of aggregates. Deficiencies of these tests are mitigated if they are supplemented by an examination by petrographers experienced in problems of concrete. However, a reorientation of testing techniques is needed urgently.

Research has developed a chemical test which quickly and reliably identifies the aggregates causing distress of concrete containing high-alkali cement. A promising technique for quickly evaluating freezing and thawing durability of aggregates and concrete and for exploring quantitatively the character of internal voids has been devised. Research on the properties that determine bond between aggregate and cement should be promoted with vigor. More must be learned about the occasionally significant properties, such as thermal expansivity and the ability to shrink and expand with drying and wetting. Efficient and practicable tests of the fundamental properties of aggregates should be developed; yet, inevitably, economy will not permit application of complex tests to all aggregates proposed for use. Thus, the petrographic examination may serve to establish the testing program necessary for each aggregate, with a view to measuring properties that cannot be estimated with sufficient accuracy.

Improvements in the testing of aggregates should proceed hand in hand with research to improve cement and techniques of mixing, placing, and curing. Only thus can the full potentialities of concrete be realized.

Acknowledgments

The research on alkali reactivity and on the relation of capillarity to freezing and thawing durability, described in this paper, was conducted by the Petrographic Laboratory, Geology Section, Research and Geology Division of the Bureau of Reclamation in Denver. Research activities of the laboratory were performed by and under the direction of Duncan McConnell, former head of the laboratory; Richard C. Mielenz, present head of the laboratory; William Y. Holland, petrographer; Kenneth T. Greene, chemist-petrographer; and N. Cyril Schieltz, X-ray crystallographer. The data presented in this paper were compiled by Messrs. McConnell and Mielenz.

APPENDIX. BIBLIOGRAPHY ON CONCRETE AGGREGATES

(1) **"The Weathering of Natural Building Stones,"** by R. J. Schaffer, *Special Report No. 18*, Dept. of Scientific and Industrial Research, Building Research, London, 1932, p. 1.

(2) **"A Working Hypothesis for Further Studies of Frost Resistance of Concrete,"** by T. C. Powers, *Proceedings*, A.C.I., Vol. 41, 1945, p. 245.

(3) **"Concrete Manual,"** Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., 1942, p. 1.

(4) **"A Concrete Failure Attributed to Aggregate of Low Thermal Coefficient,"** by J. C. Pearson, *Proceedings*, A.C.I., Vol. 38, 1941, p. 29.

(5) **"Effect of Various Coarse Aggregates Upon the Cement-Aggregate Reaction,"** by C. H. Scholer and W. E. Gibson, *ibid.*, Vol. 44, 1948, p. 1009.

(6) **"Handbook of Physical Constants,"** *Special Paper No. 36*, Geological Soc. of America, 1942, p. 31.

(7) **"Factors Affecting the Thermal Expansion of Concrete Aggregate Materials,"** by W. H. Parsons and W. H. Johnson, *Proceedings*, A.C.I., Vol. 40, 1944, p. 457.

(8) **"Petrography of Concrete Aggregate,"** by Roger Rhoades and R. C. Mielenz, *ibid.*, Vol. 42, 1946, p. 581.

(9) **"Drying Shrinkage of Concrete As Affected by Many Factors,"** by R. W. Carlson, *Proceedings*, A.S.T.M., Vol. 38, 1938, p. 430.

(10) **"Hydration-Pressure Relationships in Clays and Heaving Shales,"** by H. H. Powers, B. L. Towle, and J. B. Plaza, *Oil and Gas Journal*, Vol. 41, 1942, p. 215.

(11) **"Experiments on the Freezing of Certain Building Materials,"** by W. N. Thomas, *Technical Paper No. 17*, Dept. of Scientific and Industrial Research, Building Research, London, 1938, p. 1.

(12) **"Thermal Properties of Concrete,"** *Bulletin No. 1*, Boulder Canyon Project Final Reports, Pt. VII, Cement and Concrete Investigations, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., 1940, p. 1.

(13) **"Influence of Cement and Aggregate on Concrete Expansion,"** by T. E. Stanton, *Engineering News-Record*, February 1, 1940, p. 40.

(14) **"Concrete Deterioration at Parker Dam,"** by R. F. Blanks, *ibid.*, March 27, 1941, p. 46.

(15) **"Notes on the 'Effect of Alkalies in Portland Cement on the Durability of Concrete,' "** *Proceedings*, A.S.T.M., Vol. 43, 1943, p. 199.

(16) **"An Interesting Case of Dangerous Aggregate,"** by J. C. Pearson and G. F. Loughlin, *Proceedings*, A.C.I., Vol. 19, 1923, p. 142.

(17) **"Studies of the Accelerated Soundness Tests,"** by L. V. Garrity and H. F. Kriege, *Proceedings*, Highway Research Board, National Research Council, Vol. 15, 1935, p. 237.

(18) **"The Relation Between Los Angeles Abrasion Test Results and the Service Records of Coarse Aggregates,"** by D. O. Woolf, *ibid.*, Vol. 17, 1937, p. 350.

(19) **"Looking Inside the Abrasion Machine,"** by W. S. Cook, L. C. Gilbert, and H. F. Kriege, *Pit and Quarry*, May, 1941, p. 43.

(20) **"Petrographic Examination of Concrete Aggregates,"** by Richard C. Mielenz, *Bulletin*, Geological Soc. of America, Vol. 57, 1946, p. 309.

(21) **"Cracking in Concrete Due to Expansive Reaction Between Aggregate and High-Alkali Cement As Evidenced in Parker Dam,"** by H. S. Meissner, *Proceedings*, A.C.I., Vol. 37, 1941, p. 549.

(22) **"Cement-Aggregate Reaction in Concrete,"** by Duncan McConnell, Richard C. Mielenz, William Y. Holland, and Kenneth T. Greene, *Proceedings*, A.C.I., Vol. 44, 1948, pp. 93–128. (*a*) p. 95, Fig. 2. (*b*) p. 110, Table 3. (*c*) p. 124, Fig. 20.

(23) **"Chemical Test for Reactivity of Aggregates with Cement Alkalies; Chemical Processes in Cement-Aggregate Reaction,"** by Richard C. Mielenz, Kenneth T. Greene, and Elton J. Benton, *ibid.*, pp. 193–221.

(24) **"California Experience with the Expansion of Concrete through Reactions Between Cement and Aggregate,"** by T. E. Stanton, O. J. Porter, S. Meder, and A. Nicol, *ibid.*, Vol. 38, 1942, p. 209.

(25) **"Investigation of Causes of Delayed Expansion of Concrete in Buck Hydroelectric Plant,"** by H. A. Kammer and R. W. Carlson, *ibid.*, Vol. 37, 1941, p. 665.

(26) **"Notes on Effect of Alkalies in Portland Cement on the Durability of Concrete,"** by F. B. Hornibrook, H. Insley, and L. Schuman, *Proceedings*, A.S.T.M., Vol. 43, 1943, p. 218.

(27) **"Studies to Develop an Accelerated Test Procedure for the Detection of Adversely Reactive Cement-Aggregate Combinations,"** by T. E. Stanton, *ibid.*, p. 875.

(28) **"Alkali Etching Tests on Concrete Aggregates,"** by Willard H. Parsons and Herbert Insley, *Proceedings*, A.C.I., Vol. 40, 1944, p. 229.

(29) **"Reactivity of Aggregate Constituents in Alkaline Solutions,"** L. Bean and J. J. Tregoning, *ibid.*, Vol. 41, 1944, p. 37.

(30) **"Studies on the Alkalinity of Some Silicate Minerals,"** by R. E. Stevens, *Professional Paper 185-A*, Geological Survey, U. S. Dept. of the Interior, 1934, p. 1.

(31) **"Frost Heaving,"** by Stephen Taber, *Journal of Geology*, Vol. 37, 1929, p. 428.

(32) **"Handbook of Chemistry and Physics,"** edited by C. D. Hodgman, Chemical Rubber Pub. Co., Cleveland, Ohio, 29th Ed., 1945, p. 1.

(33) **"Some Observations Regarding Frost Action,"** by A. F. Gill and J. F. J. Thomas, *Proceedings*, Highway Research Board, National Research Council, Vol. 19, 1939, p. 281.

(34) **"The Destruction of Concrete by Frost,"** by A. R. Collins, *Journal*, Inst. C. E., Vol. 23, 1944, p. 29.

(35) **"The Mechanism of Frost Heaving,"** by Stephen Taber, *Journal of Geology*, Vol. 38, 1930, p. 303.

(36) **"A Working Hypothesis for Further Studies of Frost Resistance of Concrete,"** by T. C. Powers, *Proceedings*, A.C.I., Vol. 41, 1945, pp. 245–272. (*a*) p. 272–9.

(37) **"Report of the Building Research Board for the Year 1931,"** H. M. Stationery Office, London, 1932, p. 21.

(38) **"Deterioration of Concrete Dams Due to Alkali-Aggregate Reaction,"** by R. F. Blanks and H. S. Meissner, *Transactions*, ASCE, Vol. 111, 1946, p. 743.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

CONTROL OF THE HYDRAULIC JUMP BY SILLS

By John W. Forster[1] and Raymond A. Skrinde,[2] Juniors ASCE

Synopsis

The hydraulic jump is a phenomenon well known to hydraulic engineers as a useful method of dissipating excess energy and thereby preventing erosion below overflow spillways, chutes, and sluices. Violent turbulence inherent in the jump has also resulted in its use in mixing and aeration processes. The value of relevant data in the design of such processes has initiated many experimental investigations, so that the dimensions and characteristics of a hydraulic jump in a rectangular channel may be predicted within practical limits. Accordingly, the main difficulty in designing any structure whose functioning is dependent on the formation of a hydraulic jump lies in determining how to insure the formation of the jump and control its position, rather than in predicting its dimensions.

When the normal tailwater depth is such that supercritical flow tends to continue downstream, formation of a jump may be effected by sills placed on the channel bottom. Unless such a sill is properly designed, however, a true hydraulic jump will not form, and the desired results will not be realized. For example, a drowned jump may occur, within which the high-velocity flow does not diverge satisfactorily; the flow may simply pass over the sill as a standing wave or swell; or it may impinge upon the sill and be deflected forward and upward in a sheet or spray. In these cases the aeration or the energy dissipation, or both, may be negligible compared to that occurring in a true hydraulic jump.

The object of the study described herein was to investigate the performance of such sills in level rectangular channels, and to attempt to formulate a general method of design. The experimental results are shown to compare favorably with a theoretical analysis, and data of general application are presented dimensionlessly.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **September 1, 1949.**

[1] Engr., Hydro-electric Const. Dept., Cia. Brasileira Administradora de Serviços Tecnicos, São Paulo, Brazil.

[2] Hydr. Engr., Corps of Engrs., Walla Walla Dist., Walla Walla, Wash.

Two types of sills were investigated: First, a vertical nonaerated weir with a sharp crest, and, second, an abrupt rise in the channel bottom. Charts have been prepared which may be used to determine sill dimensions for preliminary design purposes and as a guide in analyzing or predicting the effect of such sills under given conditions.

Theoretical Development

The theory of the hydraulic jump has been adequately discussed elsewhere,[3] so that a full review here is considered unnecessary. However, a brief review of the main relationships is given as a basis for the development which follows.

In the case of water forming a hydraulic jump in a horizontal flume, the ratio of unit inertial reaction to unit gravity force is a criterion for dynamic similarity. Such a ratio is comprised in the Froude number:

$$\mathbf{F} = \frac{V}{\sqrt{g\,d}} \qquad (1)$$

—in which V and d are, respectively, the mean velocity and the depth at any section, and g is the acceleration due to gravity—so that geometrically similar cases of such flow in open channels will be dynamically similar (that is, velocities and pressures will be similarly distributed) when the Froude numbers for the two cases are the same.

Other forms of this dimensionless parameter for dynamic similarity have been used; for example,[4] the "kinetic flow factor" $\frac{V^2}{g\,d}$, or the ratio of the velocity head to the depth of flow $\frac{V^2}{2\,g\,d}$. The particular form used for open-channel flow is a matter of individual preference. Owing to its general significance, the Froude number has been used in this investigation. For the condition of critical flow, $\mathbf{F} = 1$; for tranquil or subcritical flow, $\mathbf{F} < 1$; and for shooting or supercritical flow, $\mathbf{F} > 1$.

The conditions of the jump phenomenon are such that the momentum principle may be conveniently used in analyzing the problem. As a first approximation, the assumption is made of uniform velocity distribution before and after the jump. Considering the free body bounded by the end sections, the level floor, and the water surface (Fig. 1(*a*)), the only external force effective in changing the momentum is the pressure acting at each of the end sections, where the pressure distribution is hydrostatic. Considering unit width in a rectangular horizontal channel, the net force acting in the direction of flow is equated to the increase in momentum flux between the sections; thus:

$$\frac{\gamma\,d_1^2}{2} - \frac{\gamma\,d_2^2}{2} = \rho\,q\,(V_2 - V_1) \qquad (2)$$

[3] "The Hydraulic Jump in Terms of Dynamic Similarity," by Boris A. Bakhmeteff and Arthur E. Matzke, *Transactions*, ASCE, Vol. 101, 1936, pp. 630–647.

[4] "The Hydraulic Jump in Sloping Channels," by Carl E. Kindsvater, *ibid.*, Vol. 109, 1944, p. 1109.

in which γ is the specific weight of the liquid; ρ is the density of the liquid; and q is the rate of flow per unit width. The equation of continuity may be written

$$q = V_1 d_1 = V_2 d_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Combining Eqs. 2 and 3 to eliminate q and V_2, and introducing the value of **F** for the approaching flow, leads to the dimensionless relationship:

$$\frac{d_2}{d_1} = \tfrac{1}{2}\left(\sqrt{1 + 8\,\mathbf{F}^2} - 1\right) \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

For supercritical flow on a level floor, frictional resistance dissipates the energy of the fluid, effecting a decrease in velocity and an increase in depth in the direction of flow. The water surface profile is an H3-curve.[5] At any

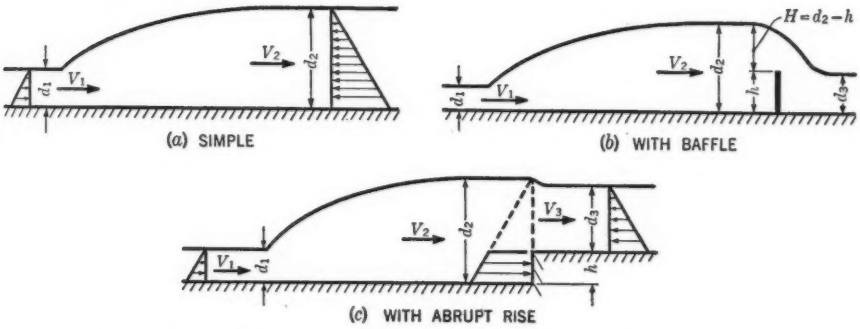


FIG. 1.—DEPTH, VELOCITY, AND PRESSURE RELATIONS IN THE HYDRAULIC JUMP

point along the profile the flow conditions are characterized by a particular value of **F**. A hydraulic jump will form where there occurs a downstream depth d_2 that satisfies Eq. 4. If the downstream depth is then decreased, the jump will recede downstream to a point where Eq. 4 is again satisfied. Similarly, if the downstream depth is increased, the jump will be forced upstream and may be drowned at the source (sluice gate, overflow spillway, or similar device).

If the downstream depth is lower than that necessary for the formation of a jump, a sharp-crested baffle or an abrupt rise in the channel bottom may be used to increase it. In either case, an analysis of the flow conditions may be made similar to that for the hydraulic jump, in that only the equations expressing the principles of continuity and momentum are necessary for a solution.

Dimensional Considerations.—In the hydraulic jump phenomenon just considered, the sequent depth d_2 is primarily dependent on the depth d_1 and the velocity V_1 of the supercritical flow, and the value of g which determines the ratio of γ to ρ. Thus:

$$d_2 = f_1(V_1, d_1, g) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Dimensional analysis then leads to the conclusion that d_2/d_1 is expressible as a

[5] "The Hydraulics of Steady Flow in Open Channels," by Sherman M. Woodward and Chesley J. Posey, John Wiley & Sons, Inc., New York and London, 1941, p. 64.

function of $\frac{V_1}{\sqrt{g\,d_1}}$, or **F**, so that

$$\frac{d_2}{d_1} = \phi_1(\mathbf{F}) \quad \ldots\ldots (6)$$

—which functional relationship was previously developed (see Eq. 4).

Similarly, if h is the height of the sill introduced into the channel to effect the formation of a jump (see Figs. 1(*b*) and 1(*c*)) then, in general, h will depend as before on V_1, d_1, and g, and also on the normal downstream or tailwater depth d_3 and the distance X from the toe of the jump to the sill. Thus:

$$h = f_2(V_1, d_1, g, d_3, X) \quad \ldots\ldots (7)$$

Dimensional analysis then leads to the conclusion that h/d_1 is expressible as a function of **F**, d_3/d_1, and X/d_1. However, since the exact position of the jump cannot be treated theoretically X/d_1 will be considered constant, with a magnitude sufficient to insure a complete jump. This simplifies the analysis so that

$$\frac{h}{d_1} = \phi_2\left(\mathbf{F}, \frac{d_3}{d_1}\right) \quad \ldots\ldots (8)$$

Analysis for a Sharp-Crested Weir.—In the case of a weir being used to effect the formation of a jump (see Fig. 1(*b*)), further simplification is possible, due to the fact that, as long as the downstream depth d_3 is less than that required to produce submergence of the crest, the flow pattern upstream from the weir is independent of d_3. Under these conditions d_3 is eliminated as a variable, so that for any given position of the jump relative to the weir, h/d_1 should be a function of **F** alone, or

$$\frac{h}{d_1} = \phi_3(\mathbf{F}) \quad \ldots\ldots (9)$$

To develop this relationship, first the momentum equation (Eq. 2) for the hydraulic jump is written as before. Then to the continuity equation for the previous case (Eq. 3) must be added a term for the discharge over the weir. For a sharp-crested, aerated weir with negligible velocity of approach, the theoretical discharge equation is

$$q = \tfrac{2}{3}\sqrt{2\,g}\,H^{\frac{3}{2}} \quad \ldots\ldots (10)$$

However, this relationship is not expected to apply in the case of a low non-aerated weir in a rectangular channel. For want of something better, empirical coefficients may be applied to the theoretical expression in order to obtain a first approximation. The continuity equation then becomes

$$q = V_1 d_1 = V_2 d_2 = C_1 C_2 C_3 (\tfrac{2}{3}\sqrt{2\,g}\,H^{\frac{3}{2}}) \quad \ldots\ldots (11)$$

in which C_1, C_2, and C_3 are coefficients defined as follows:

$C_1 = 0.61$, which is the limiting value of the Bazin suppressed weir coefficient for no velocity of approach as H approaches infinity. It applies provided the

normal downstream depth will not affect the discharge over the weir[6]—that is, $d_3 < (d_2 - \frac{3}{4}h)$.

$$C_2 = 1 + 0.55 \left(\frac{H}{d_2}\right)^2 \quad \ldots\ldots (12a)$$

which is Bazin's weir coefficient to provide for the effect of approach velocity.[7]

$$C_3 = 0.878 + 0.128 \frac{h}{H} \quad \ldots\ldots (12b)$$

which is Forchheimer's coefficient[6] to correct for nonaeration of the nappe and is valid for $H \gtrless 0.4\,h$. (This condition was found to obtain for all experimental runs during the investigation.)

Introducing these values and expressions for the coefficients (Eqs. 12) in Eq. 11 and eliminating certain variables,

$$N^2 + N = 0.664\,(N - r)^3 \left[0.770 + 0.847 \left(\frac{N - r}{N}\right)^2 + 0.234 \left(\frac{N - r}{N}\right)^4 + 0.225 \left(\frac{r}{N - r}\right) + 0.248 \frac{r}{N}\left(\frac{N - r}{N}\right) + 0.068 \left(\frac{N - r}{N}\right)^3 \frac{r}{N} + 0.016 \left(\frac{r}{N - r}\right)^2 + 0.018 \left(\frac{r}{N}\right)^2 + 0.005 \left(\frac{N - r}{N}\right)^2 \left(\frac{r}{N}\right)^2 \right] \quad \ldots (13a)$$

in which, for brevity,

$$N = \frac{d_2}{d_1} = \tfrac{1}{2}\,(\sqrt{1 + 8\,\mathbf{F}^2} - 1) \quad \ldots\ldots (13b)$$

and

$$r = \frac{h}{d_1} \quad \ldots\ldots (13c)$$

This is an implicit equation for h/d_1 in terms of **F**, and may be solved by trial. The resulting plot is shown as curve (*a*) in Fig. 2.

Interpretation of Fig. 2.—Curve (*a*), Fig. 2, indicates the theoretical relationship, completed by empirical coefficients, between the Froude number **F** of the approaching supercritical flow and the $\frac{h}{d_1}$-ratio necessary for the formation of a hydraulic jump that is complete upstream from the weir (providing it is not submerged). Coordinates lying above and to the left of the curve represent conditions in which the weir is too high, so that the jump is forced upstream and may be drowned at the source. Coordinates lying to the right of the curve represent conditions in which the weir is too low, so that the jump is forced downstream and may be washed out.

The point $\mathbf{F} = 1, h/d = 0$ corresponds to the case of critical flow discharging over an abrupt fall in the channel bottom. If the analysis were exact, the curve would be expected to pass through this point, since for the unique con-

[6] "Hydraulik," by P. H. Forchheimer, B. G. Teubner, Leipzig und Berlin, 1930, p. 382.
[7] "Hydraulics," by George E. Russell, Henry Holt and Co., New York, N. Y., 1941, 5th Ed., p. 162.

ditions of critical flow $d_1 = d_2$. However, for $\mathbf{F} = 1$, the computed value of $h/d_1 = -0.15$ indicates an inexactness in this extreme limit due to the empirical coefficients used in the analysis.

It is to be noted that there is nothing inherent in the solution which determines the distance X from the toe of the jump to the weir. The Froude number of the flow is that at the toe of the jump, and the only assumption relative to

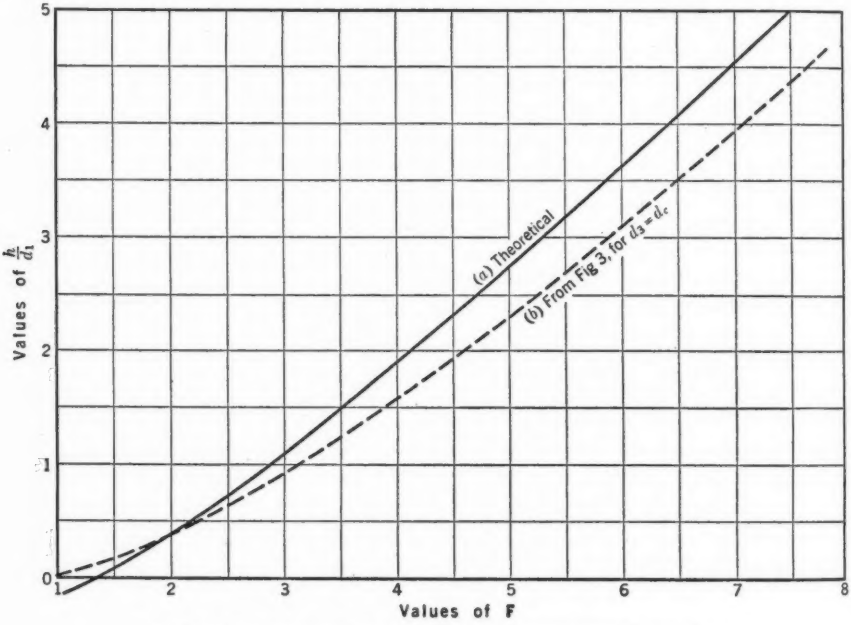


FIG. 2.—VARIATION OF RELATIVE BAFFLE HEIGHT WITH F FOR THE FORMATION OF THE HYDRAULIC JUMP

the distance X is that it be of sufficient magnitude to permit the establishment of uniform velocity distribution at the depth d_2 upstream from the weir. Variation of X/d_1 with $\mathbf{F}$ and h/d_1 must be investigated experimentally rather than theoretically.

Analysis for an Abrupt Rise.—Lowering the approach section below the level of the normal channel will result in an abrupt rise in the channel bottom which may also be used to control a hydraulic jump. This case varies from that of a weir, however, in that the flow pattern is affected by the tailwater depth d_3 for all possible conditions of flow in a level channel. Consequently (see Fig. 1(*c*)),

$$\frac{h}{d_1} = \phi_4\left(\mathbf{F}, \frac{d_3}{d_1}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and this functional relationship also may be developed by use of the momentum and continuity equations.

First, the momentum equation between sections 1 and 2 is identical with Eq. 2. By assuming hydrostatic pressure distribution at the section of the

abrupt rise, the force exerted by the face of the rise may be evaluated as $\frac{1}{2}\gamma h\,(2\,d_2 - h)$ and the momentum equation between sections 2 and 3 becomes

$$\frac{\gamma\,d^2_2}{2} - \frac{\gamma\,d^2_3}{2} - \frac{\gamma\,h}{2}\,(2\,d_2 - h) = \rho\,q\,(V_3 - V_2) \quad \ldots\ldots\ldots (15)$$

Combining with Eqs. 2 and 15, the continuity equation,

$$V_1\,d_1 = V_2\,d_2 = V_3\,d_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

and eliminating d_2, V_2, V_3 gives

$$\left(\frac{d_3}{d_1}\right)^2 = 1 + 2\,\mathbf{F}^2\left(1 - \frac{1}{d_3/d_1}\right) + \frac{h}{d_1}\left(\frac{h}{d_1} - \sqrt{1 + 8\,\mathbf{F}^2} + 1\right) \ldots (17)$$

For a fixed value of h/d_1 Eq. 17 is implicit for d_3/d_1 in terms of **F** and may be solved by trial. The resulting plot of d_3/d_1 versus **F** for certain constant values of h/d_1 is shown in Fig. 3.

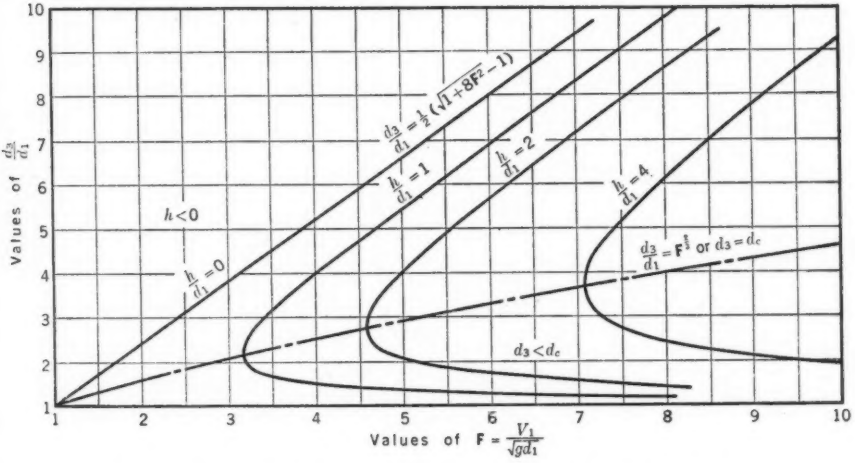


FIG. 3.—THEORETICAL VARIATION OF RELATIVE TAILWATER DEPTH WITH F AND RELATIVE HEIGHT OF ABRUPT RISE IN CHANNEL BOTTOM

Interpretation of Fig. 3.—Dividing the chart (Fig. 3) into three main regions are two lines, which pass through the intersection of the coordinate axes. By considering the conditions represented by these dividing lines, the significance of each region becomes apparent. The curve for $h/d_1 = 0$ is defined by the equation for the hydraulic jump in a level rectangular channel (Eq. 4), and thus represents equality between the normal tailwater depth and the sequent depth of the supercritical flow. Above this line, the region within which d_3 is greater than d_2 corresponds to the case in which a drop in the channel bottom (rather than an abrupt rise) is necessary to maintain a jump.

The curves of constant h/d_1 pass through a minimum value of **F**. By setting $\frac{d\mathbf{F}}{d(d_3/d_1)} = 0$, the equation of the line of minimum **F**-values is deter-

mined as

$$\frac{d_3}{d_1} = \mathbf{F}^{\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

It may be shown that this relationship corresponds to the condition that the tailwater depth is critical; that is,

$$d_3 = d_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19a)$$

or

$$\frac{V_3}{\sqrt{g\, d_3}} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19b)$$

However, as flow is not normally changed from subcritical to supercritical by a reduction in the cross section of a level channel,[8] it is obvious that if a jump forms, d_3 cannot be less than d_c. Hence, the region below the line represented by Eq. 18, within which $d_3 < d_c$, cannot represent jump conditions as assumed in the analysis, but corresponds to the case in which no jump forms, the supercritical flow shooting over the sill and continuing downstream.

Between these two dividing lines, the curves indicate the theoretical relationship between $\mathbf{F}$, h/d_1, and d_3/d_1 for the formation of a hydraulic jump that is complete just upstream from the rise. For a given abrupt rise and discharge in a channel, V_1, d_1, d_3, and h are known. The value of h/d_1 indicates the position of a curve, whereas the coordinates ($\mathbf{F}$, d_3/d_1) define a point. If this point lies to the left of the curve, the sill is too high and the jump will be forced upstream where it may be drowned; if it is to the right, it will be forced downstream toward the sill and may be washed out.

Further interpretation of the curves may be made by considering the case in which for a fixed height of rise h the depth d_1 is also fixed, as by a sluice gate, so that h/d_1 is constant. The upper branch of the corresponding $\frac{h}{d_1}$-curve then represents conditions under which a true hydraulic jump forms. As the discharge, and hence $\mathbf{F}$, is reduced, the constant $\frac{h}{d_1}$-curve indicates how the tailwater depth d_3 must be decreased to maintain a stable jump, so that d_3 approaches d_c. Further decrease in $\mathbf{F}$ will then result in the jump moving upstream and drowning the initial conditions. (At this point, when discharge takes place over the crest of the abrupt rise at the critical depth, the effect of the abrupt rise becomes essentially that of a weir. To illustrate this, the four values ($\mathbf{F}$, h/d_1) defined by the line representing $d_3 = d_c$ in Fig. 3 have been replotted in Fig. 2. The resulting curve (curve (*b*), Fig. 2) agrees closely with that for the weir, and the discrepancy may be attributed partly to the error attached to the empirical constants C_1, C_2, C_3, and partly to the fact that the assumption of hydrostatic pressure distribution becomes less justified as d_3 approaches d_c.)

The lower branch of the $\frac{h}{d_1}$-curve in Fig. 3 (below the line $d_3/d_1 = \mathbf{F}^{\frac{1}{3}}$) corresponds qualitatively to conditions in which no jump forms, the supercritical flow shooting over the sill and continuing downstream. Quantitatively this

[8] "Elementary Mechanics of Fluids," by Hunter Rouse, John Wiley & Sons, Inc., New York, N. Y., 1946, p. 137.

section of the curve will be inexact, because the assumption made regarding the pressure distribution becomes quite unwarranted under these conditions.

Laboratory Apparatus

Experiments were carried out in a rectangular flume 10 ft long, 1 ft wide, and 28 in. deep, having glass walls and a brass floor. At one end was an entrance tank into which the water flowed through a regulating valve from a constant-level tank overhead. An adjustable sharp-edged vertical sluice gate was used to regulate the depth of flow into the flume from the entrance tank. Downstream water depths were regulated by a steel tailgate, hinged at the bottom, at the outlet end of the flume. Discharge was measured by an elbow meter, which had been previously calibrated by a sharp-crested weir, and the Rehbock discharge formula. Water depths in the flume were measured by a three-pronged point gage mounted on a carriage sliding on level rails. The prongs of the point gage were so spaced that readings were taken at the half and quarter points across the flume. The gage could be read by vernier to 0.001 ft.

Brass plates from 0.048 ft to 0.348 ft in height were used as weirs. For the abrupt rise, a false bottom extended from the top of a weir to the tailgate, a distance of approximately 2.5 ft. This false bottom could be affixed flush with the top of any of the weirs, and leveled by supports and shims. Brass plates used with this arrangement varied from 0.078 ft to 0.261 ft in height.

The maximum height of sill used was limited by the available head and the discharge measuring facilities. Minimum heights were limited to magnitudes below which probable percentage errors involved in the measurements were considered too high for the data to be of value.

Experimental Procedure

For the Weir.—In the case of the sharp-crested, nonaerated weir it was decided to maintain an unsubmerged crest, as assumed in the theoretical development, in order to eliminate the tailwater effect, and hence the variable

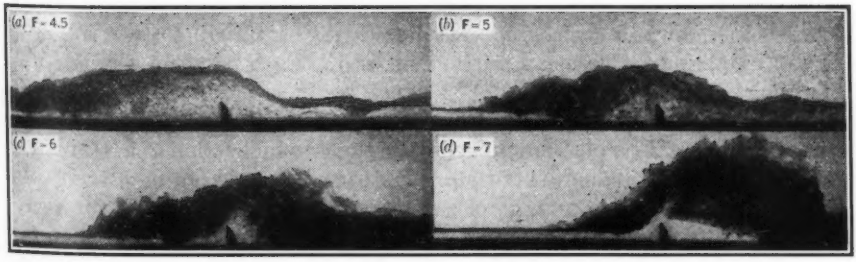


Fig. 4.—Effect of Baffle Showing Variation of Water-Surface Profile with F for $h/d_1 = 2$

d_3/d_1. The investigation was then conducted to determine the variation of the form of the water-surface profile (and hence X/d_2) with F and h/d_1.

Four values of the ratio h/d_1—$\frac{1}{2}$, 1, 2, and 4—were investigated, and for each value of h/d_1 runs were made to determine the water-surface profile for

certain fixed values of **F** within the practical range of discharge. With h and d_1 both fixed, a true jump formed well upstream from the baffle for a low discharge. With the increase in discharge, and hence in **F**, the toe of the jump was forced downstream toward the weir (Fig. 4) until the flow either shot over the weir as a sheet, or passed over the weir as a standing wave or surge, depending on the value of h/d_1. As the toe of the jump moved downstream and closer

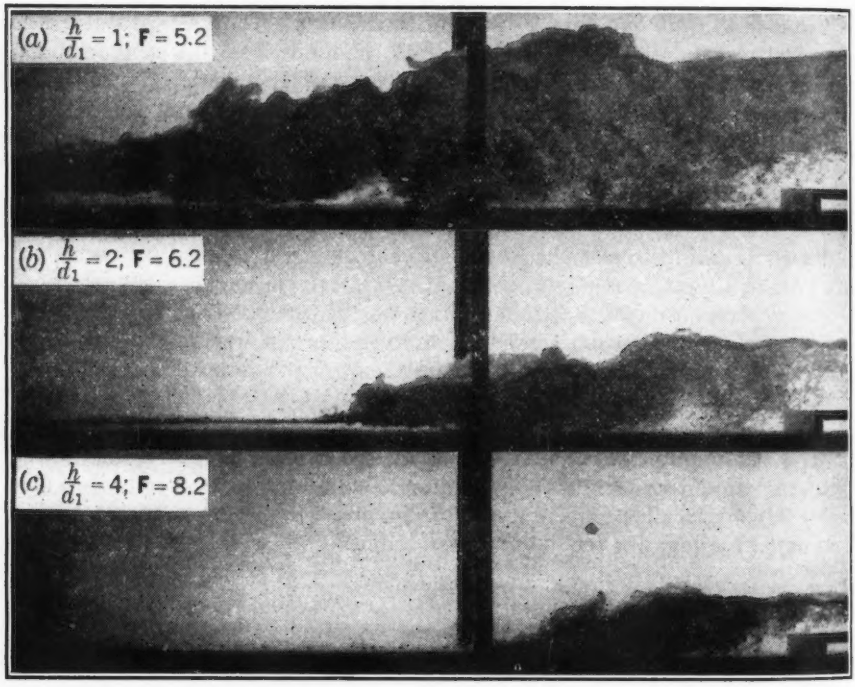


FIG. 5.—EFFECT OF ABRUPT RISE, SHOWING VARIATION OF h/d_1 WITH F FOR $d_3/d_1 = 5.5$

to the weir, the water surface became more highly agitated, so that the difficulty of obtaining a true mean profile increased. Each point on a profile was determined by averaging three independent readings, one taken for each of the three prongs on the point gage. Each individual reading was taken by setting the pointer at a position judged to be the mean elevation of the fluctuating water surface over a period of time. For each value of h/d_1, runs were made using weirs over as wide a range of size as was practicable. Thus, for every one of the four values of h/d_1 several independent profiles were obtained for each of several fixed values of **F**.

For the Abrupt Rise.—As previously explained, tailwater effects (and hence the variable d_3/d_1) were eliminated in the case of the weir by maintaining an unsubmerged crest, under which conditions the variation of the relative position of the jump with **F** and h/d_1 was determined. In order to investigate the field as fully as possible in the limited time available, it was decided, in the case of the abrupt rise, to maintain the relative position of the jump constant (thus

eliminating the variable X/d_2), and to investigate rather the effects of the tailwater. Accordingly, for the abrupt rise, the investigation was conducted to determine the variation of d_3/d_1 with **F** and h/d_1 for the condition that $X = 5\,(h + d_3)$. This value of X was selected because it seemed to provide sufficient distance for the jump to be completed before the rise, and because the tranquil flow conditions downstream from the rise permitted quite accurate measurements of $h + d_3$. Practically, this resulted in a jump, the toe of which was slightly greater than $4\,d_2$ upstream from the rise.

For any given abrupt rise, h/d_1 was fixed by regulating the sluice gate opening. The discharge, and hence **F**, was fixed by regulating the inlet valve. Then, by adjusting the tailgate, the toe of the jump was made to occur at a distance $5\,(h + d_3)$ upstream from the rise. For this condition, values of d_1, d_3, and the discharge were recorded, and values of d_3/d_1 and **F**, later computed. The minimum discharge under which such a jump formed occurred when the tailgate effect was removed, and the discharge took place as free fall at the outlet end of the flume. (Fig. 5 illustrates conditions in which $d_3/d_1 = 5.5$ for values of h/d_1 of 1, 2, and 4.)

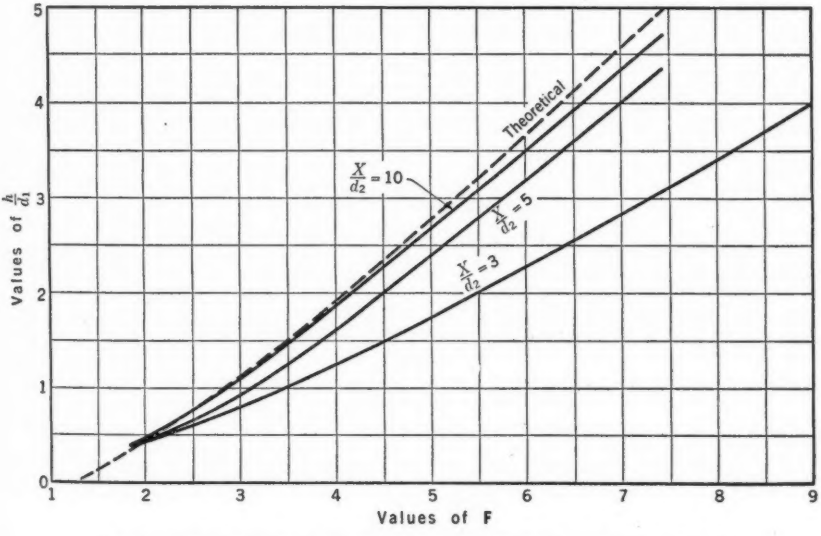


FIG. 6.—EXPERIMENTAL VARIATION OF RELATIVE BAFFLE HEIGHT WITH F AND THE RELATIVE POSITION OF JUMP

Throughout the experiments the depth measurement used to determine the recorded value of **F** was made at a distance $20\,h$ upstream from the sill. The point so defined was convenient in that it was fixed for any given baffle, and lay upstream from the toe of all but the longest jumps.

EXPERIMENTAL RESULTS

For the Weir.—For fixed values of h/d_1 and **F**, the corresponding water surface profile was obtained by using d_1 as the unit of length and so that all recorded points could be plotted dimensionlessly for several independent runs.

Consequently, for each of the four values of h/d_1—$\frac{1}{2}$, 1, 2, and 4—profiles for four or five values of **F** were available, varying from the case in which the jump formed well upstream from the weir to that in which the high-velocity flow shot over the baffle either as a spray or as a standing wave. Fig. 4 illustrates the sequence of profiles for $h/d_1 = 2$.

For the sake of brevity these plotted profiles have not been included, only the summarized results being presented. Thus, for each of the four values of h/d_1 investigated the value of **F** corresponding to three values of X/d_2 (specifically, 3, 5, and 10) was determined from the plotted profiles by interpolation. The resulting curves are shown in Fig. 6, where the theoretical relation from Fig. 2 is also plotted to provide a comparison with experimental results.

For the Abrupt Rise.—All experimental results are for the condition that $X = 5\,(h + d_3)$. Fig. 7 shows the variation of d_3/d_1 with h/d_1 and **F** for values of h/d_1 of $\frac{1}{2}$, 1, 2, and 4.

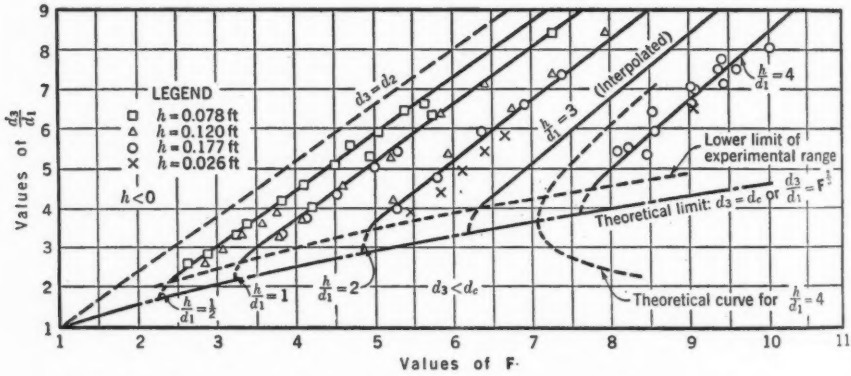


FIG. 7.—EXPERIMENTAL VARIATION OF RELATIVE TAILWATER DEPTH WITH F AND THE RELATIVE HEIGHT OF ABRUPT RISE

COMPARISON OF RESULTS WITH THEORY

For the Weir.—The relation of the theoretical function of h/d_1 versus **F** to the curves plotted from observed data is indicated in Fig. 6. All the experimental curves lie to the right of the theoretical, but approach it more closely as the ratio X/d_2 increases. This is to be expected because, as the distance X increases, the velocity distribution of the flow just upstream from the baffle approaches more closely the uniformity assumed in the theoretical development.

Also, it is obvious that the increment between the curves for $X/d_2 = 3$ and $X/d_2 = 5$ is much greater than that between the curves for $X/d_2 = 5$ and $X/d_2 = 10$. This occurs because the velocity distribution closely approaches uniformity for $X/d_2 = 5$, any further increase in X/d_2 having little additional effect. Thus, the curve for $X/d_2 = 10$ marks the closest approach that may be expected of an experimental curve to the theoretical. The remaining discrepancy must be attributed to inexactness of the theoretical development due to the empirical coefficients used, and to lack of agreement between the experimental conditions and the idealized ones of the theory.

For the Abrupt Rise.—On comparing the experimental $\frac{h}{d_1}$-curves (Fig. 7) with those of the theoretical development (Fig. 3), it is apparent that within the range of experimental data all the curves closely approximate straight lines, the slopes of which are similar in both cases. This is as would be expected, as the experimental results serve simply to fix the position of the curves for a certain relative position of the jump—namely $\frac{X}{h + d_3} = 5$.

However, the experimental curve for any value of h/d_1 is seen to be shifted somewhat to the right of the corresponding theoretical curve. This shift is almost negligible for $h/d_1 = 1$, but preceding toward the right of the chart, as the values of h/d_1 increase, the amount of shift between the corresponding experimental and theoretical curves increases. The extent of the maximum discrepancy within the experimental range, for the curve $h/d_1 = 4$, is indicated in Fig. 7.

It may be shown analytically that, as the ratio $\frac{X}{h + d_3}$ increases (corresponding to a velocity distribution approaching uniformity at the section of the abrupt rise), the experimental curves shift to the left toward the theoretical curves. Had the experimental runs been conducted with the ratio $\frac{X}{h + d_3}$ equal to 10, then, as in the case of the weir, the experimental curves would approach the theoretical more closely.

In Fig. 7 it is also apparent that the dashed line defining the lower limit of the experimental range lies above the line defining the theoretical limit. This is because the laboratory apparatus did not permit reducing the tailwater depth to the critical depth; otherwise, the lower limit of the experimental range would be shifted downward.

Practical Application of Results

In practice, d_1 and V_1 of the supercritical flow, d_2, and d_3 may be computed. Then, provided $d_3 < d_2$, a sill may be used to prevent the hydraulic jump from being forced downstream. With these factors known, Figs. 6 and 7 are useful for the purpose of either predicting the effect of a given sill on the flow pattern or of determining the height of sill required to control a jump.

For the Weir.—Fig. 6 permits an analysis of the effect of a given weir for known supercritical and tailwater flow conditions, provided that the tailwater depth does not affect discharge over the crest—that is $d_3 < (d_2 - \frac{3}{4} h)$. In any given case, the coordinates (**F**, h/d_1) indicate a unique point on the diagram. If this point lies between the experimental curves, hydraulic jump action will result, the corresponding interpolated value of X/d_2 indicating the relative position of the jump. If the point lies above the curves, the jump will be forced upstream; when drowning results, the weir will serve to deflect any high-velocity bottom currents. If the point lies below the curve $X/d_2 = 5$, a shortened, incomplete jump or spray action, will result; only a qualitative prediction of the effect is possible, depending on the distance of the point from the given curves.

For design purposes, it is proposed that the curve for $X/d_2 = 5$ in Fig. 6 be used, under conditions of maximum discharge. Then, for maximum discharge the weir will insure the formation of a jump, whereas for any lesser discharge the weir will serve to create a stilling basin in which a longer or a drowned jump occurs.

The case of the submerged weir has not been investigated quantitatively. Ordinarily, unsubmerged conditions prevail over a wide range of discharge; if submergence occurs, the jump is forced upstream with possible drowning as a result.

For the Abrupt Rise.—Fig. 7 provides a method of predicting the performance of a given abrupt rise when V_1, d_1, d_2, d_3, and h are known. Three situations are possible, as follows:

First, if the point ($\mathbf{F}$, d_3/d_1) lies above the line $h/d_1 = 0$, the conclusion is that $d_3 > d_2$, so that an abrupt rise would serve only to increase the drowning effect. Consequently, a drop in the channel bottom rather than an abrupt rise is necessary to maintain a jump. This case has not been investigated in this study.

Second, if the point ($\mathbf{F}$, d_3/d_1) lies within the experimental range (that is, between the lines for $d_3 = d_2$ and $d_3 = d_c$), then the position of the point relative to the corresponding $\frac{h}{d_1}$-curve indicates the effect of the abrupt rise on the flow pattern. Thus, as was pointed out earlier, if the point lies on the corresponding $\frac{h}{d_1}$-curve, a jump will form with $\frac{X}{h + d_3} = 5$; if it lies to the right of the curve, the sill is too low, so that the jump will be forced downstream toward the abrupt rise and may be washed out; if it is to the left of the curve, the sill is too high and the jump will be forced upstream and may be drowned.

Third, if the point ($\mathbf{F}$, d_3/d_1) lies in the region below the line $d_3 = d_c$, then the normal downstream flow is supercritical—that is, the slope is steep for the given discharge so that the control section is at the abrupt rise. Formation of a jump then results in discharge taking place over the crest of the abrupt rise at the critical depth, so that the case is analogous to that of the weir. Fig. 6 may be used, therefore, as outlined in the preceding section.

Fig. 7 may be used to determine the necessary length and depth of stilling basin when V_1, d_1, and d_3 are known. For most tailwater rating curves, the critical condition—that is, that requiring the maximum value of h—occurs for conditions of maximum discharge. It is proposed, then, that the point ($\mathbf{F}$, d_3/d_1) be first defined for conditions of maximum discharge, and the corresponding value of h/d_1 determined by interpolation. The effect of the sill for lesser flows should then be determined from the figure as previously outlined, and if necessary the sill height increased. For the expected range of discharge, this procedure will indicate the minimum height of sill necessary to prevent the jump from being washed out. For the critical condition a jump will form, and be complete upstream from the sill, whereas for other discharges a proportionally longer or drowned jump will occur; in the latter case the sill will serve to deflect any high-velocity bottom currents.

When the downstream slope is steep, and hence the tailwater depth is less than critical, either a weir or an abrupt rise of the same height may be used. Consequently, Fig. 6 may be used for design purposes.

Conclusions

For any supercritical discharge in a rectangular channel, when the corresponding d_2 is greater than d_3, a hydraulic jump may be created and prevented from receding downstream by either a weir extending across the channel or an abrupt rise in the channel bottom. When the normal downstream depth is less than critical, then an abrupt rise and a weir of the same height are interchangeable.

For conditions in which the tailwater does not affect discharge over the crest, a theoretical analysis, based on the momentum principle and completed by empirical coefficients, indicates a definite relationship between the depth and velocity of the shooting flow and the height of a sharp-crested, nonaerated weir necessary to produce a jump. A similar theoretical analysis indicates a definite relationship between V_1, d_1, d_3, and the height of an abrupt rise necessary to produce a jump for conditions in which $d_3 > d_c$.

Data from experimental investigations conducted in a level rectangular flume verify these theoretical relationships within close limits, and any discrepancies may be accounted for logically. The experimental data are presented in dimensionless charts—one for an abrupt rise, corresponding to the outlet of a depressed stilling basin, and one for a low, sharp-crested, nonaerated weir—that provide a method of predicting qualitatively the effects of such sills on supercritical flow. These charts serve also to indicate the type of hydraulic jump control applicable under given conditions, and may be used in the design of small structures, or in the preliminary design of large structures by indicating the range of sill sizes to be investigated in detail in specific model studies.

Acknowledgments

This paper has been prepared from a thesis submitted by the writers in partial fulfilment of the requirements for the degree of Master of Science, in the Department of Mechanics and Hydraulics, in the Graduate College of the State University of Iowa at Iowa City. The experimental tests were made in the Hydraulics Laboratory of the Iowa Institute of Hydraulic Research at Iowa City. Grateful acknowledgment is given to Hunter Rouse, M. ASCE, for suggestions and guidance during the investigation and for constructive criticism during the writing of the paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE RELATIONSHIP BETWEEN PILE FORMULAS AND LOAD TESTS

Discussion

BY ROBERT D. CHELLIS

ROBERT D. CHELLIS,[29] M. ASCE.—Valuable points that should be kept in mind when arriving at decisions regarding pile capacities have been presented in the discussion of this paper.

Mr. Lee calls attention to tests involving clay strata. Because of the greatly restricted use of formulas, if soils with some clay or silt strata were excluded, a number of cases having mixed but predominantly granular soils were included. In some such cases the ratios of formula to test loads were above 100% and in other cases below, with no trend definitely apparent. By consideration of foot-by-foot driving records and borings, the engineer should judge the effects of the clay strata and make such allowance as he deems proper for this condition.

Mr. Lee states that it has been recognized in England for the last 10 years that the Hiley type of formula is the only one having any reasonable claims to fairly accurate forecasts of the ultimate supporting value of piles. The writer is glad to have this fact again called to the attention of engineers in the United States. Mr. Lee feels that too much attention has been directed to showing the inaccuracy of formulas that are known to be entirely unsuitable for the purposes to which they have been applied. This is true in England, but in the United States the Engineering-News formula is still in practically universal use and is contained in most building codes. It was published again in the influential new building code for New York, N. Y., thoroughly revised and adopted on November 17, 1948. To shake this trust, actual evidence is needed, such as is produced in the paper. Admiral Angas' discussion bears witness to the fact that the Engineering-News formula and its variants "die hard" in the United States.

NOTE.—This paper by Robert D. Chellis was published in May, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1948, by Donovan H. Lee; November, 1948, by Jacob Feld, Richard F. Kafka, G. B. R. Pimm, and George D. Hartley; and December, 1948, by W. Mack Angas.

[29]Structural Engr. Stone & Webster Eng. Corp., Boston, Mass.

There are two aspects to the problem of why the Hiley type of formula may not have given even better agreement with test results. One is the question of theoretical accuracy of the formulas and measurement of values for use in the formulas, and the other, mentioned both by Mr. Lee and Mr. Feld, is the question of the accuracy and interpretation of load test results. Investigation of the theoretical correctness of the Hiley type formulas was considered beyond the scope of this paper. The writer's concern was with results. Regarding load tests, the graphical results plotted in Fig. 1 and Fig. 2 are based on the values reported for the test loads. The tests were performed under a variety of conditions, by different engineers, and were of various durations. Failure points were picked by various persons who doubtless used different rules for interpreting these values. Data were not available to put all test results on a common basis, and no doubt if they had been some differences in results would have appeared. Nevertheless, the results do represent the general type of information now obtained in the United States and, because of the number of the tests, should give general trends for the various types of formulas, piles, and hammers. It is hoped that the use of accurate driving readings and load tests to failure will become universal, and that better data will be published and become available for study.

Mr. Feld indicates the time factor in settlement and questions the accuracy of load test results. This factor has a considerable effect. In tests performed by the writer, he endeavors to allow each increment of load to come to rest without settlement for a day, thus obtaining a more accurate load settlement curve than is usual. Most of the tests reported were made by others, however, and the time factor was frequently not reported. There are many rules in use for selecting the failure point for test loads, giving quite a difference in results. A number of such rules were published for comparative study in 1946.[11]

Mr. Feld and Mr. Pimm mention the distribution of resistance between end bearing and surface friction. It is regretted that the methods commonly used in England for driving a mandrel in a shell arranged to drive the shell and tip together—or, alternately, to obtain total driving resistance and end resistance separately—are not yet in general use in the United States, where very few contractors are equipped to perform this operation.

The writer read with interest Mr. Kafka's article[25] proposing a driving formula containing terms for elastic losses. With regard to the application of driving formulas to cohesive soils, however, it would seem that the apparent elastic properties of such soils during resistance to a dynamic blow might not be the same as the elastic properties present over a long time. If clay were like rubber, it would possess one set of definite elastic properties. Pile driving in cohesive soils, however, often meets with high resistance to quick displacement of the contained water, and also creates a zone of temporarily remolded material and a layer of water around the pile that temporarily cannot be forced into the soil. This water forms a lubricating film next to the pile so that neither the transfer of stress value between the pile and soil, nor the bearing

[11] "Making and Interpreting Pile Load Tests," by Robert D. Chellis, *Engineering News-Record*, June 13, 1946, pp. 914–919.

[25] "Die Theorie der Pfahlgruendungen," by Richard F. Kafka, J. Springer, Berlin, 1912.

value of the soil, acting during the blow, are values that have a known relationship to the pile-bearing value. Both Mr. Kafka's formula and the Hiley type of formulas have terms for elastic rebound of the soil and thus assign an expenditure of a proportion of the driving energy to the soil. This deduction from energy is desirable, but would seem to be more accurate in the case of cohesionless soils than in the case of cohesive soils. There is the further consideration, mentioned by Mr. Feld, that load tests of cohesive soils often indicate safe values that prove excessive in the actual structures. It is to be supposed that indicated dynamic resistances on piles would be subject to the same consideration.

Mr. Pimm suggests that the weight of ground moved downward by a pile during driving be included in the term for pile weight in the formulas. This condition probably occurs to some extent with all piles, but particularly with H-piles for which the writer previously has advocated the inclusion of the weight of soil between the flanges.[10] This soil has elastic properties different from those of the pile; but probably the modulus of elasticity, even if it were known, is small relative to that of the pile, and may be ignored if an elastic loss is computed. When the value of rebound is measured in the field, this value is taken into account.

Mr. Pimm states that experience has shown that frictional resistance is a very capricious factor. The test results presented by Mr. Hartley illustrate this statement. Driving conditions similar to those given by Mr. Hartley have just been noted by the writer on a friction pile project on the west bank of the Hudson River opposite New York City.

Admiral Angas doubts that stresses in a pile during driving may be determined with reasonable accuracy by the Hiley formula. The writer was able to observe the driving of 566 pine piles with varying tip diameters and cross-sectional areas to various sets, and to measure rebounds for use in the Hiley formula.[8] The small percentage of breakage was reasonably constant for the specified minimum sets for the various tip diameters, as determined by computed stresses, rising quickly when the assumed limiting value of fiber stress was exceeded. This method of determining driving limits for wood piles extending through water or soft material to short penetration in a bearing stratum has been used by the writer on other projects and has allowed driving with practical avoidance of breakage. The piles with the larger tips can withstand hard driving compared to those with smaller tips, the problem being the determination of the stress and avoidance of stresses exceeding the yield point. With this type of bearing and wood piles, it is the stress near the lower end that becomes critical, because of the small diameter at this point.

The writer has also had opportunity to compute stresses in H-piles and fluted steel shell piles by the Hiley formula, and has observed failure of the metal at the resistances giving a computed fiber stress equal to the yield point.[10]

The writer's experience agrees with the observations of Admiral Angas in the San Diego experiment, with heavier piles requiring smaller sets to reach a given tip elevation.

[10] "Pile-Driving Handbook," by Robert D. Chellis, Pitman Pub. Corp., New York, N. Y., 1944.

[8] "Field Determination of Damaging Stresses During Driving," by Robert D. Chellis, *Engineering News-Record*, May 30, 1946, pp. 863–865.

No method of computation of pile capacities, whether by dynamic formulas or measurement or computation of stresses at various points in the pile, would seem to be correct unless the same load-carrying capacity were indicated for piles of the same dimensions regardless of weight and material of their composition, when driven by the same hammer in the same soil to the same tip elevations. The capacity of the soil to support load would not seem to be affected by the material of which the pile is composed inside its bounding dimensions.

The writer thanks all discussers of the paper, and hopes that much further material on the correlation of driving and test results will be made available to the profession by others, so that more final conclusions can ultimately be reached and be accepted as working tools.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

A SIMPLIFIED METHOD OF ANALYZING SUSPENSION BRIDGES

Discussion

BY HUI-SIEN LIU

HUI-SIEN LIU.[16]—Stresses in suspension bridges under static load could be analyzed with accuracy after the deflection theory was introduced. Further effort has been aimed at two opposite ends:

(1) To eliminate the assumptions made in the deflection theory for a more accurate analysis and

(2) To make more approximations for the simplification of formulas used in practical design. The author is to be congratulated in having achieved the latter end successfully.

The author finds that the relation between the horizontal component of cable stress and the length of an advancing load under extremely simplified conditions can be fitted by a sine curve; then he concludes that the same sinusoidal relation holds for all conditions. This is expedient but not rigorous. The magnitude of error inherent in the formulas thus derived is not established unless results are compared with those produced by other accurate methods; and, in particular, the author does not prove the applicability of the principle of superposition although he has repeatedly used it.

To demonstrate this point,[17] consider a general case of loading such as that in Fig. 10. The load on the cable is equal to the tension in the suspenders which consists of two parts: The dead load, w, and the increment, s, created by the live load and change of temperature. The deflection of the cable is usually considered to be equal to the deflection of the stiffening truss. Consequently, the suspender tension corresponding to a given cable deflection, η, is defined as

NOTE.—This paper by Ling-hi Tsien was published in September, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1949, by Pedro B. J. Gravina, and G. S. Ramaswamy.

[16] Formerly Prof., Structural Eng., Tsinhua Univ., Peiping, China.

[17] "On the Theory and Design of Suspension Bridges," by Hui-Sien Liu, *Research Bulletin*, Tsinhua Univ., Peiping, China, 1946.

follows:

Span (see Fig. 10)	Cable deflection	Load on cable
Main	η	$w + s$
Side 1	η_1	$w_1 + s_1$
Side 2	η_2	$w_1 + s_2$

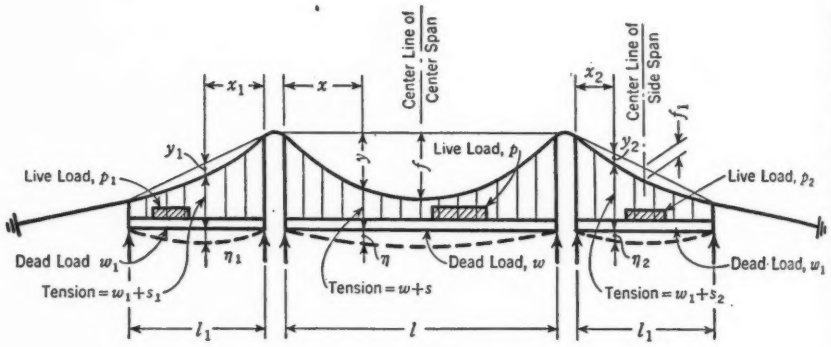


FIG. 10

Isolating the cable as a free body, the increase of strain energy in the cable, resulting from live load and temperature change, must be equal to the work done by the loads on the cable. Hence an equation of energy can be written as

$$\left(H_w + \frac{H}{2}\right)\frac{H\,L_s}{E_c\,A_o} \pm \left(H_w + \frac{H}{2}\right)\epsilon\,\Delta t\,L_t = \int_0^l \left(w + \frac{s}{2}\right)\eta\,dx + \int_0^{l_1}\left(w_1 + \frac{s_1}{2}\right)\eta_1\,dx + \int_0^{l_1}\left(w_1 + \frac{s_2}{2}\right)\eta_2\,dx \quad \ldots\ldots (128a)$$

or

$$\left(1 + \frac{\beta}{2}\right)\beta\,\frac{H^2_w\,L_s}{E_c\,A_o} \pm \left(1 + \frac{\beta}{2}\right)H_w\,\epsilon\,\Delta t\,L_t = \int_0^l \left(w + \frac{s}{2}\right)\eta\,dx + \int_0^{l_1}\left(w_1 + \frac{s_1}{2}\right)\eta_1\,dx + \int_0^{l_1}\left(w_1 + \frac{s_2}{2}\right)\eta_2\,dx \quad \ldots\ldots (128b)$$

In the second term of the left-hand member, the plus sign (+) is for an increase, and the minus sign (−) for a decrease, in temperature.

To generalize, let the live load, p, be any kind of load which can be represented by a Fourier series as

$$p = \sum_m \frac{4\,w}{m\,\pi} K_m \sin\frac{m\,\pi\,x}{l} \quad \ldots\ldots (129a)$$

in which

$$K_m = \frac{m\,\pi}{4\,w}\frac{2}{l}\int_0^l p \sin\frac{m\,\pi\,x}{l}\,dx \quad \ldots\ldots (129b)$$

Then, by methods[2,17] described elsewhere, the following series for η and s can be written

$$\eta = \frac{32}{\pi^3} f \sum_m \frac{K_m - \beta \sin^2 \frac{1}{2} m \pi}{m^3 (1 + \beta + m^2 \mu)} \sin \frac{m \pi x}{l} = \frac{32}{\pi^3} f \sum_m A_m \sin \frac{m \pi x}{l} \ldots (130a)$$

and

$$s = p - E I \frac{d^4 \eta}{dx^4} = \beta w + \frac{4 w}{\pi} (1 + \beta) \sum m^2 A_m \sin \frac{m \pi x}{l} \ldots (130b)$$

in which

$$A_m = \frac{K_m - \beta \sin^2 \frac{1}{2} m \pi}{m^3 (1 + \beta + m^2 \mu)} \ldots\ldots\ldots\ldots\ldots\ldots (131)$$

In Eqs. 130 and 131, β is the ratio between the additional and the initial horizontal components of cable stress:[2]

$$\beta = \frac{H}{H_w} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (132a)$$

and μ is the "stiffening factor" of main span, a dimensionless constant adopted to denote the degree of stiffening:

$$\mu = \frac{E I}{H_w} \left(\frac{\pi}{l} \right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (132b)$$

An expression, μ_1, for the stiffening factor of side spans can be written as

$$\mu_1 = \frac{E I_1}{H_w} \left(\frac{\pi}{l_1} \right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (132c)$$

The integrals on the right-hand side of Eq. 128*b* can then be evaluated as follows:

$$\int_0^l \left(w + \frac{s}{2} \right) \eta \, dx$$

$$= \int_0^l \left(1 + \frac{\beta}{2} \right) w \eta \, dx + \int_0^l \frac{2 w}{\pi} (1 + \beta) \eta \sum_m m^2 A_m \sin \frac{m \pi x}{l} \, dx$$

$$= \frac{64}{\pi^4} \left(1 + \frac{\beta}{2} \right) w l f \sum_{1,3,5,\ldots} \frac{A_m}{m} + \frac{32}{\pi^4} (1 + \beta) w l f \sum_{1,2,3,\ldots} m^2 A^2_m$$

$$= \frac{512}{\pi^4} \left(1 + \frac{\beta}{2} \right) n H_w f \sum_{1,3,5,\ldots} \frac{A_m}{m} + \frac{256}{\pi^4} (1 + \beta) n H_w f \sum_{1,2,3,\ldots} m^2 A^2_m$$

$$= \frac{512}{\pi^4} \left(1 + \frac{\beta}{2} \right) n H_w f \left(\sum_{1,3,5,\ldots} \frac{A_m}{m} + \frac{1 + \beta}{2 + \beta} \sum_{1,2,3,\ldots} m^2 A^2_m \right) \ldots\ldots (133)$$

[2] "The Stiffness of Suspension Bridges," by S. Timoshenko, *Transactions*, ASCE, Vol. 94, 1930, p. 377.

Writing similar expressions for the side span integrals and substituting into the energy equation of the cable,

$$\beta \frac{H_w L_s}{E_c A_o} + \epsilon \Delta t L_t$$

$$= \frac{512}{\pi^4} n f \left(\sum_{1,3,5,\cdots} \frac{A_m}{m} + \frac{1+\beta}{2+\beta} \sum_{1,2,3,\cdots} m^2 A^2{}_m \right)_{\text{main}}$$

$$+ \frac{512}{\pi^4} n_1 f_1 \left(\sum_{1,3,5,\cdots} \frac{A_m}{m} + \frac{1+\beta}{2+\beta} \sum_{1,2,3,\cdots} m^2 A^2{}_m \right)_{\text{side 1}}$$

$$+ \frac{512}{\pi^4} n_1 f_1 \left(\sum_{1,3,5,\cdots} \frac{A_m}{m} + \frac{1+\beta}{2+\beta} \sum_{1,2,3,\cdots} m^2 A^2{}_m \right)_{\text{side 2}} \quad \cdots\cdots (134)$$

The series, $\sum_{1,2,3,\cdots} m^2 A^2{}_m$ represents the effect of variations in the tension in the suspenders. This effect is sufficiently small to be neglected in the deflection theory. The series, $\sum_{1,3,5,\cdots} \frac{A_m}{m}$ converges rapidly—that is, all terms $m > 1$ can be omitted without sacrificing much accuracy. Therefore,

$$\beta \frac{H_w L_s}{E_c A_o} \pm \epsilon \Delta t L_t$$

$$= \frac{512}{\pi^4} n f (A_1)_{\text{main}} + \frac{512}{\pi^4} n_1 f_1 [(A_1)_{\text{side 1}} + (A_1)_{\text{side 2}}]$$

$$= \frac{512}{\pi^4} n f \frac{(K_1)_{\text{main}} - \beta}{1+\beta+\mu} + \frac{512}{\pi^4} n_1 f_1 \left[\frac{(K_1)_{\text{side 1}} - \beta}{1+\beta+\mu_1} + \frac{(K_1)_{\text{side 2}} - \beta}{1+\beta+\mu_1} \right]$$

$$= \frac{512}{\pi^4} \frac{n f}{1+\beta+\mu} \left\{ (K_1)_{\text{main}} + \frac{n_1 f_1}{n f} \frac{1+\beta+\mu}{1+\beta+\mu_1} [(K_1)_{\text{side 1}} + (K_1)_{\text{side 2}}] \right\}$$

$$- \frac{512}{\pi^4} \frac{n f}{1+\beta+\mu} \left(1 + 2 \frac{n_1 f_1}{n f} \frac{1+\beta+\mu}{1+\beta+\mu_1} \right) \beta \quad \cdots\cdots (135)$$

Multiplying through by $\frac{\pi^4}{512} \frac{n f}{1+\beta+\mu}$ and rearranging,

$$\beta \left(1 + 2 \frac{n_1 f_1}{n f} \frac{1+\beta+\mu}{1+\beta+\mu_1} + \frac{\pi^4}{512} \frac{H_w L_s}{E_c A_o} \frac{1+\beta+\mu}{n f} \pm \frac{\pi^4}{512} \frac{\epsilon \Delta t L_t}{n f} \right)$$

$$= (K_1)_{\text{main}} + \frac{n_1 f_1}{n f} [(K_1)_{\text{side 1}} + (K_1)_{\text{side 2}}] \mp (1+\mu) \frac{\pi^4}{512} \frac{\epsilon \Delta t L_t}{n f} \quad \cdots (136)$$

Let[4]

$$N_s = 2 \frac{n_1 f_1}{n f} \frac{1+\beta+\mu}{1+\beta+\mu_1} \quad \cdots\cdots (137a)$$

$$N_c = \frac{\pi^4}{512} \frac{H_w L_s}{E_c A_o} \frac{1+\beta+\mu}{n f} \quad \cdots\cdots (137b)$$

$$N_t = \frac{\pi^4}{512} \frac{\epsilon \Delta t L_t}{n f} \quad \cdots\cdots (137c)$$

and

$$D_{\pm} = 1 + N_s + N_e \pm N_t \quad \ldots\ldots\ldots\ldots\ldots\ldots (137d)$$

Then,

$$\beta = \frac{(K_1)_{\text{main}} + \frac{N_s}{2}(K_1)_{\text{side 1}} + \frac{N_s}{2}(K_1)_{\text{side 2}} \mp (1+\mu) N_t}{D_{\pm}} \quad \ldots\ldots (138)$$

Eq. 138 demonstrates that the principle of superposition applies to β for different load combinations only when N_s, N_t, and N_e are constants. The quantity N_t is a constant, as shown by Eq. 137*c*; but, although N_s and N_e involve β they are small in comparison to unity and a wide variation of β does not affect their values materially. It follows that the principle of superposition is applicable, in practice, in determining the horizontal component of cable stress for different load combinations.

For ordinary bridges, β varies roughly from zero for no live load to 1.0 g for full live load. An average value of $\beta = 0.5\,g$ can be assumed in Eqs. 137*a* and 137*b* without causing any serious error, thus producing the simple formulas:

$$N_s = 2\,\frac{n_1 f_1}{n f}\,\frac{1 + 0.5\,g + \mu}{1 + 0.5\,g + \mu_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (139a)$$

and

$$N_e = 0.190\,\frac{H_w L_s}{E_c A_c}\,\frac{1 + 0.5\,g + \mu}{n f} \quad \ldots\ldots\ldots\ldots\ldots\ldots (139b)$$

The expression for K_m defined by Eq. 129*b* can be determined easily for any loading. Referring to Fig. 4, let k_1 be expressed as $k'\,l$ and $k_2 - k_1$ as $k\,l$. For the general case of uniform load shown in Fig. 4,

$$K_m = \frac{m\,\pi}{4\,w}\,\frac{2}{l}\int_{k'l}^{(k+k')l} p \sin\frac{m\,\pi\,x}{l}\,dx = \frac{m\,\pi}{4\,w}\,\frac{2}{l}\left(-\,p\,\frac{l}{m\,\pi}\cos\frac{m\,\pi\,x}{l}\right)$$

$$\times \int_{k'l}^{(k+k')l} = \frac{p}{w}\sin\tfrac{1}{2}\,m\,(k + 2\,k')\,\pi \sin\tfrac{1}{2}\,m\,k\,\pi$$

$$= g \sin\tfrac{1}{2}\,m\,(k + 2\,k')\,\pi \sin\tfrac{1}{2}\,m\,k\,\pi \quad \ldots\ldots\ldots\ldots\ldots\ldots (140)$$

Using $k\,l$ to denote length of load and $k'\,l$ to denote position of load, K_m for concentrated loads and special cases of uniform loading can be readily deduced from Eq. 140. It is evident from Eq. 129*b* that K_m for the combination of a number of loads is equal to the sum of the K_m-values for the individual loads. Once again, the applicability of the principle of superposition is demonstrated.

Application of the general formula, Eq. 138, for H to the author's ten cases of loading results in a series of equations that can be compared with the author's formulas in Table 1. Doing so, the writer found that only the discrepancies for N_s and S are significant. The more accurate of the two procedures remains to be tested by checking each against the exact formula (Eq. 134) for various bridges. However, the writer believes either set of formulas will yield results sufficiently accurate for ordinary design purposes.

The author expresses the characteristics of stiffening trusses in hyperbolic functions (Part II). With a good table of hyperbolic functions at hand, the

computation required by the author's method is probably the least among all the methods. With the simplified formulas for H, he is able to formulate the conditions of maximum values for different functions which are usually found by trial. These achievements, demonstrated in the paper, will certainly facilitate the analysis considerably. However, the writer does not believe that it is justifiable to determine the section of maximum moment and the loading for it simultaneously because the equations (see Eqs. 72 and 75) are not exact and their solution is not simple. It would be more simple and dependable to determine the maximum moments at a few points near the quarter point and then to pass a curve through them.

The method of trigonometrical series as applied to the functions of the stiffening truss has an advantage over the deflection theory—that is, the general nature of the formulas. A single formula can be written for a function at any section of stiffening truss for all cases of loading. Conditions of maximum can be established in the method of trigonometrical series[17] too.

Summarizing the foregoing discussion, conclusions can be drawn as follows:

1. To derive the simple formulas for H, the writer's suggestion of using the trigonometric series[17] is more general and rigorous.

2. The author's formulas for truss functions require least computation whereas the formulas in trigonometrical series have the advantage of generality.

3. Contrary to the general impression of engineers, the analysis of suspension bridges is no longer more complicated than for other continuous structures.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

INDUSTRIAL WASTES
A SYMPOSIUM

Discussion

BY A. M. QUATTLEBAUM

A. M. QUATTLEBAUM,[16] Assoc. M. ASCE.—Because of the rapidly increasing industrialization of the southern states the problem of industrial wastes is becoming more and more important to the peoples of every state, and the lawmakers of individual states are becoming increasingly conscious of this important problem.

Professor Bloodgood has presented the problem in a concise and interesting manner, and it is significant that conclusions 2, 3, and 4 of his paper have been included in a bill proposed to the General Assembly in South Carolina in February, 1949.

The General Assembly of 1948 created a committee of three members for the purpose of "investigating and determining the proper and effective use of the fresh water resources of the state." This committee was composed of one representative of the state board of health, one representative of industry, and one representative of the South Carolina Wildlife Federation. In the letter of transmittal of a proposed bill to the General Assembly the committee stated that they had worked harmoniously together and as a result of the investigation had reached unanimous agreement that the major factors to be considered in the proper utilization of the state's waters were:

1. The promotion and maintenance of public health;
2. The conservation of the state's valuable wild life resources; and
3. The development and maintenance of industries in the state.

In the investigation South Carolina was divided into eleven areas representing the major drainage areas. The total population discharging sewage into these streams is 1,208,000 with a sewage population equivalent to 701,000. The industries of the state discharge a sewage population equivalent to 1,614,000.

NOTE.—This Symposium was published in September, 1948, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1949, by S. Leary Jones.

[16] Chf. Engr., Harllee Quattlebaum Cons. Co., Florence, S. C.

This shows that at the present time industrial wastes amount to approximately 2.3 times the domestic sewage.

The proposed bill for the control of the waters of South Carolina creates a water control authority consisting of five members—namely, one member from each of the following: The Cotton Manufacturer's Association of South Carolina, the pulp and paper industries, the South Carolina Wildlife Federation, and the Municipal Association of South Carolina (the fifth member being at all times the State Health Officer of South Carolina who is to be chairman of the group).

The authority is charged with the control of the waters of the state and is instructed to classify all rivers as to usage and to establish the standards of purity to be maintained in all cases. It shall be allowed to classify certain streams as industrial streams and in such cases the industries using said streams shall be ordered to take only such precautions as are pertinent to the health of the public. From this it can be seen that the authority is to have far-reaching powers and this section of the bill is certain to cause considerable debate.

The writer considers the bill as drawn to be far from perfect; however, he does consider it very significant that persons of such diversified interests have worked harmoniously together and it is believed that such cooperation could do much toward a better understanding and a more profitable solution to industrial waste problems everywhere.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE SIGNIFICANCE OF PORE PRESSURE IN HYDRAULIC STRUCTURES

Discussion

BY L. F. HARZA

L. F. HARZA,[54] M. ASCE.—The writer appreciates very much the several contributions to completeness and clarity offered by the discussions, and also the revelation that his case had not been clearly presented in all respects. The discussions vary from almost unequivocal endorsement to the other extreme.

Messrs. Carlson and Davis have revealed the fact that the pores in cement paste (after setting) are much smaller than the writer had assumed; in fact they are of the general magnitude expressed in a few millionths of an inch instead of hundreths. The writer is especially indebted to Mr. Riegel for calling attention to something the writer considered obvious, but which apparently was not—namely, that the ideal columns are everywhere (except where pores interrupt) in intimate contact (or continuous) with adjacent cylinders so that stresses of shear, tension, and compression may be transmitted through these contacts without disturbing the structural integrity of the concrete. Of course, these cylinders are purely imaginary subdivisions of the material by imaginary lines and surfaces. They can have no more effect on its strength than can the imaginary meridians on the earth's surface make it fall apart like sections of an orange. Imaginary lines, for the purpose of discussion, obviously leave the concrete unchanged in tensile, shear, or compressive strength. Similar failure to distinguish between imaginary division of concrete into elementary units and actual physical dissection appears to form the basis of Mr. Meyer's statement that "* * * when the author applies his reasoning to concrete, he introduces a concrete lacking in all tensile strength." There is no other possible explanation for such an interpretation. Messrs. Leliavsky and Hill also are

NOTE.—This paper by L. F. Harza was published in the December, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1948, by William P. Creager, J. S. Kendrick, Adolf A. Meyer, E. Montford Fucik, A. C. R. Albery, and W. H. R. Nimmo; September, 1948, by John S. Cotton, Serge Leliavsky Bey, A. H. Davison, R. E. Ballester, John S. McNown, and V. T. Boughton; October, 1948, by Ross M. Riegel, and Douglas McHenry; November, 1948, by Frederick L. Hotes, R. W. Carlson and Raymond E. Davis, and W. G. Morrison; December, 1948, by Raymond A. Hill, and John V. Spielman; and February, 1949, by Paul E. Raes.

[54] Cons. Engr. and Pres., Harza Eng. Co., Chicago, Ill.

under a similar misapprehension that the writer, in making the statement that there are enough pores in each small differential in elevation in the concrete mass to form a full projected area of pores, actually moves these pores into one plane and thus destroys the strength of the concrete or (in the words of Mr. Hill): "* * * a concrete dam * * * composed of materials heavier than water cannot be supported entirely by water."

All these gentlemen greatly flatter the writer by endowing him with such occult powers as to change the physical nature of concrete from what it actually is by mere process of thought; but the better conclusion is that the writer has not made himself clear and that more explanation is needed and perhaps from a different viewpoint.

Messrs. Riegel and McHenry assert that the writer "seems to distrust experimental evidence" or that he has "objections to the experimental method." Nothing could be further from the truth. The writer is not antagonistic to experimental tests, but merely claims that they are only corroborative, after the investigator has first developed a complete understanding of the phenomenon.

The only exception taken to the Leliavsky tests was that, because of believed inherent inaccuracies in principle, they did not prove that the uplift area is not 100%. The McHenry tests[43] first came to the writer's attention in June, 1948, after publication of his paper, and obviously could not have been anticipated and properly "welcomed" in the paper. These tests, however, are now most welcome indeed.

Under the heading, "Experimental Evidence," the writer gives Professor Terzaghi full credit for the experimental determination of substantially 100% uplift area reported in his paper[7] and expresses surprise and regret that the profession in the ten years that elapsed had not generally accepted Professor Terzaghi's conclusions. Prior to the appearance of the Terzaghi paper in 1936, the writer himself had held the orthodox view that 100% uplift area could not be true because of the large proportion of the section that must be in contact to support the load.

The tests,[7] in which Professor Terizaghi was unable to compress a naked cylinder of concrete significantly by subjecting the liquid in which it was submerged to very high pressure, were the beginning and inspiration of the writer's paper in June, 1936, which was substantially completed that year. In 1936 the writer accepted the fact of practically 100% internal pressure area, equivalent to 100% uplift area, as thus demonstrated experimentally, and sought a conception of the structure of concrete which would explain this demonstrated but apparently anomalous fact. The reason for a proved fact can usually, and should always, if possible, be visualized as it aids in its acceptance and understanding. Thus the hen and egg sequence in this case began with Professor Terzahgi's research, and resulted in a theory in explanation of his results.

The effect of Mr. McHenry's discussion is to deny the writer his right to use these tests in support of his theory. The Terzaghi tests demonstrated, within the limits of experimental accuracy, that the internal pressure in the pores of

[43] "The Effect of Uplift Pressure Upon the Shearing Strength of Concrete," by Douglas McHenry, *Transactions*, 3d International Cong. on Large Dams, Stockholm, Sweden, 1948 (publication pending).

[7] "Simple Tests Determine Hydrostatic Uplift," by Karl Terzaghi, *Engineering News-Record*, June 18, 1936, p. 872.

a concrete cylinder was sufficient to oppose fully the external pressure on top and bottom. Thus, in order to resist external compressive forces it was necessary for the internal forces opposing them to be exerted over the same area as the external forces—that is, over 100% of the cross section. This justifies the use of the Terzaghi tests in support of the writer's theory for it is the basis of that theory. No ordinary section of the specimens, and no theory of minute contacts, could explain to the writer's satisfaction how this could be possible with 98%, 99%, or even 91% of pore area, leaving only from 1% to 9% of area for transmitting the load.

It has been shown that enough pores for a full projected area of pores occur in concrete at very close intervals which, by the revelation of Messrs. Carlson and Davis, would be at intervals of a few thousandths of an inch at most. To be sure, such a complete cross section of pores could not be incorporated in any plane section or in any smoothly undulated surface. The pores would obviously be staggered in position or elevation, and to incorporate such a complete pore section would require the surface to dodge vertically up and down through solid material in an erratic manner. That such a surface can logically be assumed to exist cannot be questioned, however; in fact, many such sections would exist at intervals of only a few thousandths of an inch.

In order to make the naked Terzaghi specimens practically incompressible by water pressure it was necessary that these many surfaces of 100% pores be under hydrostatic pressure, fully opposing each other, and the external pressure —with only solid, poreless material, subject to comparatively negligible compressive strain, intervening between these surfaces. Even with this assumption a small compressive strain reflecting the modulus of elasticity of the solid mineral particles or gel would be expected to occur; and this may explain the 99%. However, 99% is the same as 100% experimentally and requires the same explanation.

The Terzaghi experiments were conducted with high pressure and therefore practically with uniform internal pressure. However, if such a specimen were merely submerged in water with the top at the water surface (for simplicity), each of these numerous, internal, 100% area surfaces would be subjected to a pressure equal to its depth of submergence, instead of the nearly uniform high pressure imposed by Professor Terzaghi; and the intervening solid material would be in a buoyant state to which the writer applies the term "internal buoyancy" as self-explanatory. This is a logical, direct deduction from the Terzaghi tests. "Buoyancy" is defined by Webster as "The upward force exerted upon an immersed or floating body by a fluid." The definition does not require that the force be exerted only on the bottom. In the present application it would be defined as "The internal upward force exerted upon an immersed body or portions thereof by a fluid"; and "buoyant" would be defined as "subjected to the force of buoyancy."

Thus a submerged specimen of concrete resting on the bottom is not dependent on the penetration of water pressure along the bottom contact to produce buoyancy of the specimen; such a specimen is fully buoyant in its smaller subdivisions as the result of internal pressure (shown by Fig. 5), considered to be applied to elementary columns with a pore at the top and the

bottom, or applied to the disk-like volumes between successive complete, projected areas of pores.[54a]

Messrs. Terzaghi[7] and McHenry[43] have proved, within experimental accuracy, that the uplift acts on 100% of the area and Mr. Leliavsky, in the writer's opinion, has not proved otherwise. Thus the reader is at liberty if he wishes to consider the writer's analysis as an explanation of a demonstrated fact (which in truth was the sequence of its evolution) rather than as a deductive proof. Then if he does not like this explanation he can seek another more to his liking. The writer has been unable to find a better or, in fact, any other explanation.

The writer's thesis is the end result of a process of thinking on the subject of uplift represented by Fig. 34, corresponding with the following stages of evolution:

(*a*) Not many years ago the existence of uplift was vigorously denied by eminent authorities (no illustration).

(*b*) The second stage of evolution was to concede uplift over the proportion of the area occupied by pores in a plane surface of intersection, as in Fig. 34(*b*). Thus, if concrete contained 12% voids, uplift was conceded over only 12% of the area.

(*c*) The third stage of evolution was to introduce a rough or undulating surface, as in Fig. 34(*c*), and speculate upon the percentage of pore section that could be incorporated in such a surface. This has resulted in the now orthodox assumption of uplift of over one-half and two-thirds area.

(*d*) The writer's thesis carries the evolution one stage further, as in Fig. 34(*d*).

[54a] Correction for *Transactions:* In Fig. 5, as published in December, 1947, *Proceedings*, on p. 1514, the pore at the bottom of column B should be only the upper half of the circle.

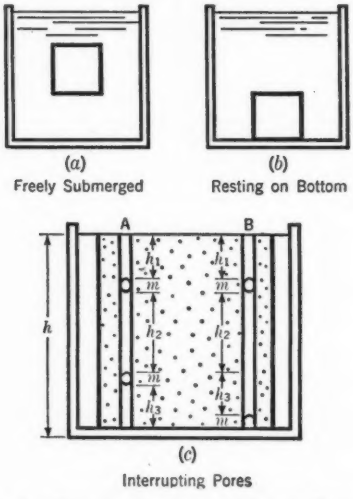


FIG. 5.—EFFECT OF INTERNAL VERSUS BOTTOM PORES ON BUOYANCY

The only new question introduced by the writer is how rough a rough surface is permitted to be in order to incorporate additional pore area without invalidating the theory that uplift could be exerted in the pores of such a surface. Who can say where the limit should be and by what theory or principle of judgment it would be determined? The only limit, in the writer's opinion, would seem to be nature's limit of roughness beyond which no more pore area could be incorporated in the assumed surface. It will be shown that, if it is rough enough, it can incorporate 100% of pore area as in Fig. 34(*d*).

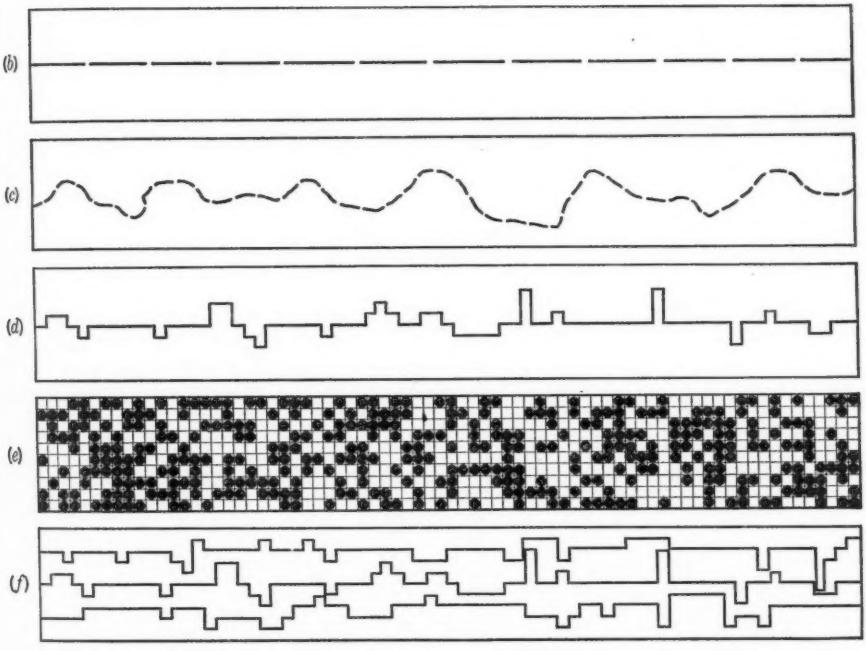


FIG. 34

This curve is platted from Fig. 34(*e*), which shows the result of rolling a seventy-four-ball "Bingo" game ten times, stopping each time that thirty-seven balls had been turned and platting their numbers represented by black dots in the corresponding squares of cross section paper containing seventy-four squares per line. Thus, each line is half-filled with black dots indicating pores chosen by chance, very crudely simulating, in two dimensions only, cement paste with 50% voids.

In Fig. 34(*f*) three lines have been shown through approximately adjacent 100% surfaces, the middle one being identical with Fig. 34(*d*). Starting from the median horizontal line there is platted in Fig. 34(*d*) the line of minimum departure from that line which will pick up a full projected area of black squares or voids. This procedure is adopted to show, with as little dependence on the imagination as possible, how rough a rough surface might need to be to incorporate 100% projected area of pores in cement. To do so, of course, re-

quires the surface to dodge up and down through solid material between pores. Although this surface appears very rough, the actual dimensions are microscopic; such surfaces occur in close proximity in actual concrete and are separated in fact by only a few thousandths of an inch.

To visualize more easily both the theory of a complete projected area of pores and the theory of elementary cylinders, Fig. 35 shows a small patch of a concrete section with contact areas between particles of great importance—not microcontacts by any means. This picture was made by a friend as his idea of the miscrostructure and without knowledge of its intended use. Therefore, it is fortuitous as far as the writer is concerned The pore area is also much less than 50% of the total, as in cement, thus making the problem more difficult.

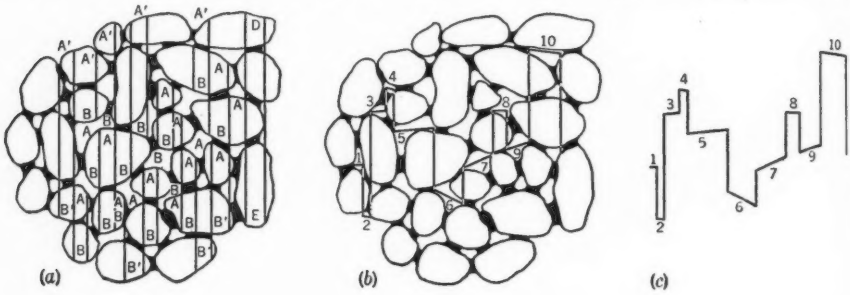


FIG. 35

The big hurdle (which is difficult to clear in understanding this theory) is the belief that large areas of contact on the horizontal exposure of particles is necessary to support the load. This is true, but it is also true that these areas do not interfere with the buoyancy of the particles. It has previously been stated that a column erected through such a horizontal area of contact would eventually terminate in a pore at the top and the bottom and would thus be equivalent to ab in Fig. 1.

Fig. 35(*a*) has been subdivided by projecting all horizontal areas of contact upward and downward until they terminate in a pore at the top and the bottom or reach the limits of the sketch. The top of each such column is marked A and the bottom, B. At some places the limit of the sketch introduces an uncertainty as to how soon a pore would be encountered if the sketch were extended. Such points are marked A′ or B′ and in some places subdivision is not necessary. In fact, it is only necessary where horizontal contacts occur. One column, marked DE, encounters three horizontal contacts and no pores to the limits of the specimen; but it is reasonably assumed that, if the specimen extended further, pores would be encountered at the top and the bottom.

A careful study of Fig. 35(*a*) will show that the entire area has been, or can be, so subdivided that these horizontal contacts are absorbed, so to speak, and become a part of a column with top and bottom pore, and that the entire area, ignoring uncertainties around the border, becomes a system of vertical cylinders or columns all in exactly the same position as column ab in Fig. 1, and therefore buoyant if submerged and the pores filled with water under pressure corresponding with the depth of each pore below the water surface.

More important than the foregoing is the fact that this pictured specimen is strong; it is a solid in the sense that any cement mortar or concrete is solid; the horizontal contacts that support the load are still there and yet they do not interfere with the buoyancy of the newly conceived cylindrical units. This is what the writer as been "harping on" for twelve years without perhaps having previously presented as clear a picture to help the imagination.

Likewise from the other standpoint of a full projected area of pores subject to buoyancy the writer has repeated Fig. 35(*a*) as Fig. 35(*b*) with the vertical cylinders omitted and with a staggered line resembling Fig. 34(*d*), chosen so that all the horizontal or sloping lines are within pores. Fig. 35(*c*) shows this staggered surface in detached position to indicate how closely it resembles the general form of Fig. 34(*d*). The horizontal and sloping lines are numbered to correspond with their respective pores.

In submerged concrete the elementary cylinders in Fig. 35(*a*), like ab in Fig. 1, are subjected to buoyancy by reason of the difference in depth of the top and the bottom below the water surface; or, if the reader prefers, the entire disk-like volume between two contiguous surfaces as in Fig. 34(*f*) or in Fig. 35(*c*) may be considered as collectively buoyant because each surface comprises a complete 100% projected area of voids.

To the foregoing argument some will state that Fig. 34(*d*), or Fig. 35(*c*), is an extreme and fantastic form of surface to assume. True enough, and it is easier, for the writer at least, to visualize the elementary cylinders; but it cannot easily be denied that such surfaces, incorporating complete projected areas of pores, actually exist at frequent intervals in the mass symbolically represented by the three surfaces in Fig. 34(*f*). If they exist the designer has a right to consider what the uplift force on such a surface would be. It would obviously be the hydrostatic pressure over 100% of the projected area of the specimen, and the specimen above this surface is not floating on water and without strength. In a submerged body of concrete the weight of that part above such an assumed surface would be reduced by buoyancy.

The conception, expressed by several discussers, that acceptance of 100% uplift area necessitates a weak concrete with no area left to support the load even to the point of requiring a structure "floating on water" seems to be the difficult "hurdle" to pass in an understanding of this problem; but the reverse is really the case as compared with other theories. Thus, the elementary cylinders visualized by the writer overlap each other with the pores at the top and the bottom of adjacent cylinders at different elevations and (substantially in the words of Mr. Riegel) are everywhere in intimate lateral contact with adjacent cylinders, permitting shear, tension, and compression to be transmitted through these contacts without disturbing the structural integrity of the concrete. (The word "cylinder" is substituted advisedly for Mr. Riegel's term "prism" because a cylinder is defined by Webster as "A surface generated by a line moving parallel to itself," and is therefore general, whereas a prism applies only to flat faces—a special case.)

These lateral contact areas are not limited by the writer's theory and may constitute in magnitude a large proportion of the section of the specimen. In

fact, the original horizontal contact areas remain intact to support the load, which therefore does not presuppose weakness of the concrete.

On the contrary, any theory based on the belief that the load must be transmitted through minute points of contact and that the surface of uplift is an irregular surface which passes almost entirely through microvoids must necessarily assume a strength of these minute contacts far beyond any known possible strength of mineral substance. Thus an uplift area of 99% on this theory must assume that 1% of the area carries the load and that the strength of this contact is 100 times as great as the composite strength of concrete, thus requiring for 4,000-lb concrete a strength of minute contact points of 400,000 lb per sq in. Even the 91% of water contact and 9% of solid determined by Mr. Leliavsky would require a tensile strength of mineral particles of $9 \times 4{,}000$, or 36,000 lb, which is equal to the elastic limit of mild steel.

By the writer's theory it is therefore quite reasonable to believe in exactly 100% uplift area, whereas by the older theory it is very difficult if not impossible to accept even 91%, and definitely impossible to accept from 97% to 99%, of area. In fact, to be acceptable, these high areas of uplift require exactly the same explanation, in the writer's opinion, as 100%.

Professor Raes tells the story beautifully when he states:

> "Under such conditions [the writer's elementary cylinders] it is evidently needless to imagine threads of astounding tensile strength when n_w approaches unity, and the value $n_w = 1$ is easily explained."

On the other hand, Mr. Leliavsky declares that the assumed surface must be a potential surface of rupture "which is physically possible." This would seem to be a very reasonable condition to impose if actual rupture enters into the application of the phenomenon to a practical problem (which it does not, in uplift or buoyancy problems in concrete). In general, the buoyant effect is less than the weight, at least in concrete, and the question of fracture on such a surface does not enter.

Nevertheless, assume that such a surface does fracture. A tension rupture on such a ragged surface as shown in Fig. 34(*d*) or Fig. 35(*c*) is conceivable only with a similar tensile stretch of the imagination, and a shear failure along such a surface is inconceivable by any stretch of the imagination. It would appear then that such a surface cannot satisfy Mr. Leliavsky's criterion.

If an assumed failure section through the specimen were to cut across some of these high peaks or low valleys in the 100% line (and it is logical to believe that even a tension failure, and certainly a shear failure, would do so), the uplift from pore pressure on the resulting surface would, by common acceptance, be effective over only that part of the area of the section composed of pores.

Reference to the McHenry test[43] would seem to prove otherwise. These were triaxial shear tests from which the uplift area was determined to be 100% within the limits of experimental accuracy. These tests appear to be a very valuable contribution to the literature and it is unfortunate that they were not published in the United States where they could be more generally accessible and better understood and evaluated.

Although the writer is not familiar in detail with the tests beyond public information,[43] it is known that they are designed to determine the effect on the

shearing strength of concrete resulting from variable pore pressure across the shearing surface obtained by nitrogen gas. One can be certain without inquiry that these shear failures did not break along a surface of 100% pore area, such as that in Fig. 34(*d*). In fact, one can be sure that almost poreless aggregate pieces were fractured in large proportion and that the pore area incorporated in the fracture was very much less than 100%; and yet the tests apparently indicate that the pore pressure had the effect, in the Coulomb equation, of weakening the specimens in shear to the same extent, by reduction in pressure across the surface of fracture, as would have been the case by pore pressure over 100% of the surface of fracture. Apparently irregular surfaces of 100% pore area, similar to those in Fig. 34(*f*), are so close together that the entire mass behaves under test merely like a lighter material as a whole, and therefore as one having 100% uplift over the entire surface of fracture even though much of this fracture is through solid material. Now that this has apparently been proved, it does not seem strange, although the writer would not have ventured such a prediction. (Incidentally, this is an important concession on the writer's part to the value of research.)

Thus it would appear that fracture follows the weakest section including aggregate pieces, where such pieces are weaker without pores than the cement paste with approximately 50% pores; but evidently the surface determining the effect of pore pressure on strength will be the surface incorporating the greatest pore area. Thus the two surfaces are determined by two different factors, one by physical weakness and one by pore area. Concrete therefore behaves with reference to shear fracture, when subjected to pore pressure, as if this pore pressure were exerted over 100% of the area. It would seem even more likely to do so under tensile stresses. The writer is greatly indebted to the Bureau of Reclamation and to Mr. McHenry for this proof.

One of the most common points raised in opposition to the writer's theory is that the concrete is too dense for the water pressure to penetrate except near the water surface. Consider only horizontal joints which are known as likely to develop seepage rapidly. In his paper the writer referred only briefly to the fact (in the fourth paragraph following Eq. 12) that only joint pressure needs to be considered; but Mr. Fucik has shown more clearly in his discussion that it is necessary only to consider joint seepage. His discussion requires some amplification. Referring to Fig. 9(*b*), assume that the elementary vertical cylinders extend from line AB to line DE (two successive construction joints), instead of extending only from pore to pore. The difference in pressure in the two joints is represented by line C′ B′ EF. From D to B′ the difference in pressure head is constant and equal to the height of the columns. From point B′ to point E the pressure in joint DE reduces to zero but so do the elementary columns which, in this section terminate in air at the face of the dam. The difference in pressure is still equal to the height of these columns. The principle of buoyancy of columns is still correct if each column terminates in a pore or pores in the construction joint at the top and bottom of a column. Since only microscopic departure from the nominal surface of the construction joint is necessary to incorporate a full projected area of pores, it would not seem questionable that pores close to the joint will reflect the pressure existing in the joint and therefore existing on the top or bottom of an elementary column.

In fact, it is necessary only to assume two surfaces like those in Fig. 34(*d*), one in each construction joint, whereas it would be reasonable to expect many, as in Fig. 34(*f*).

Furthermore, it is necessary to consider that only one horizontal construction joint is subject to seepage such as AB, Fig. 9(*b*), instead of two. The joint pressure at point A is h and at point B is zero. A straight-line reduction between points A and B obviously results in full uplift of the assumed triangular dam if the area over which the pressure is effective is 100%. As before, it will be 100% on a rough surface incorporating a full projected area of pores.

Thus one may assume the entire body of the concrete as saturated, which is equivalent to assuming contiguous surfaces as in Fig. 34(*f*). On the other hand, one may assume elementary vertical cylinders between these contiguous surfaces, only the construction joints to be under pressure with like surfaces perhaps 5 ft apart or only one such surface in one construction joint or the foundation contact. The net results are the same in all cases—that is, uplift equivalent to buoyancy of the concrete. The entire theory herein presented is necessary to support the existence of 100% uplift at construction joints even if impervious concrete between joints were to be conceded.

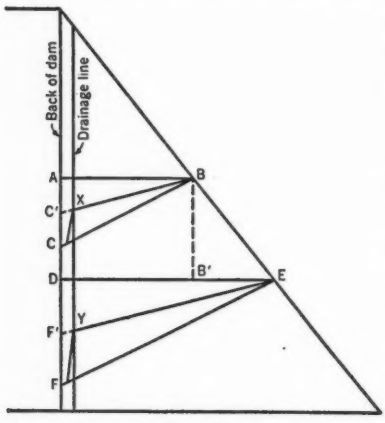


FIG. 36

Many of the points raised in discussion are believed to have been already answered herein, but a few remarks about individual discussions are in order and will be made in the order of appearance of the discussions in *Proceedings*, and only when not previously answered.

It is orthodox in these days to assume, as Mr. Creager does, that drainage near the back of a dam will reduce the uplift. The rule he gives is seemingly rational when considering any one horizontal section alone; but the important fact which deserves recording is that the reduction in magnitude of uplift pressure, if it occurs at all at adjacent joints, does not necessarily reduce the uplift force materially, if at all. This is because uplift is a differential force analogous to buoyancy, and the buoyancy of column ab, Fig. 1, would not be increased or decreased by a difference in the depth of submergence.

Thus refer to Fig. 36, which is similar to Fig. 9(*b*), with similar triangles ABC and DEF representing uplift at two elevations in the dam, precisely as in Fig. 9(*b*). It has been shown that the differential pressure between construction joints AB and DE is represented by the difference in the pressure lines CB and FE and is constant and equal to the difference in elevation of the sections or an uplift equal to full buoyancy.

Now suppose that drainage wells were to be provided near the back of the dam which would tend to reduce each pressure to one-half magnitude at the line of wells while remaining at full value at the back line of the dam as in lines CXB and FYE, much as recommended by Mr. Creager. Lines XB and YE

would still be parallel, or substantially so. The differential pressure between elevations AB and DE would still be constant and equal to their difference in elevation; and therefore the lines indicate full buoyancy between the two elevations AB and DE to point B′ directly under B. From point B′ to point E it would be one-half buoyancy; but on 5-ft lifts this would be, perhaps, only 4 ft of a thickness of dam of the general magnitude up to 300 ft or 400 ft and thus would be negligible, disappearing entirely in considering only differential elevations.

The same conclusion applies for any other uniform, proportional reduction in uplift magnitude at two near-by elevations. Thus Mr. Creager's drainage analysis, theoretically, applies only if no more than one construction joint or rock contact is under pressure. In order to accomplish its purpose, drainage would have to be increasingly effective with increasing depth. For example, assume that drainage were to prove totally ineffective at elevation AB, thus maintaining the full static pressure line CB, and that it were to prove one half effective at elevation DE, thus reducing the pressure diagram to FYE, Fig. 36. Then the uplift diagram would be the area between line CB and line YE, indicating a reduction in buoyancy (uplift) near the back, where this result is most needed, gradually increasing to full buoyancy near the face. Therefore, drainage of the foundation, or drainage into the concrete near the base, is more effective than drainage for the entire height. Since drainage by wells accomplishes only partial relief, as in Fig. 36, its value, if any, is greatly overrated. To be fully effective, instead of wells, there should be a complete layer of drained, porous material near the back of the dam which will drain all the water entirely or there should be a sheet metal diaphragm to exclude it, as suggested by the writer in 1934.[2] The latter would preferably be a sheet rolled in a dovetailed section like some roof or floor metal now available, and of stainless steel or of other similar rust-resisting metal.

Messrs. Kendrick (Fig. 12) and McNown (Fig. 23(*a*)) have both shown flow nets for the simplified triangular dam with water at the apex, which the writer uses as the basis of analysis. These differ very greatly. Since there can be only one correct flow net, the writer would select that of Professor McNown, because Mr. Kendrick's net does not satisfy the important boundary condition that the drop in velocity potential (piezometric head) between adjacent lines of constant potential must be a constant. The pressure is atmospheric along the uppermost streamline, and therefore a constant increment in piezometric head can be obtained only if the increment of elevation is constant, a condition satisfied only by Professor McNown. This also requires correction of the uplift diagram ABU, Fig. 12, which then becomes triangular.

Mr. Albery admits full uplift on horizontal surfaces such as in a gravity dam, but limits lateral pressure on vertical surfaces parallel to lines of seepage flow to the percentage of porosity, say, 12%. In the ideal dam, with all seepage proceeding at right angles to the axis, the writer would exclude it entirely as the pressure would be equal on both sides of such a surface. Professor McNown has shown (Fig. 27) that it does exist where monolith joints divert seepage laterally. The pressure on vertical planes perpendicular to

[2] Discussion by L. F. Harza of "Stability of Straight Concrete Gravity Dams," by D. C. Henny, *Transactions*, ASCE, Vol. 99, 1934, p. 1085.

the lines of flow is the effect described by the writer as the progressive application of the water load through the structure instead of on only the back of the dam, each decrement in seepage pressure representing an increment in load. The writer cannot agree with Mr. Albery's Eq. 14 and maintains that Eq. 13 is the correct one for a bending moment about C due to pore pressure at section BC. However, it is not a complete solution of the bending moment about C since external forces and their moments about C would need to be included when considering the part of the wing above line BC as a free body. This was introduced merely to illustrate one prominent instance of false conventional reasoning.

The writer does not condemn this type of dam; in fact he considers it a very good one. He believes that none of the precautions indicated by Mr. Albery, nor the fears expressed by Professor Ballester, are justified. The writer once had occasion to examine such a dam nearing completion. The rapid external cooling had formed cracks along the cylindrical surfaces of the buttress heads and parallel to the axis of same, and a space of 1 in. existed between adjacent buttress heads, this space being provided with a copper seal. This situation was obviously very dangerous but was corrected merely by filling the 1-in. spaces. It was found that any spreading force due to pressure in the cracks or pores was thus resisted against the adjacent buttress heads, an ample precaution.

Mr. Cotton's report of the condition of the Manchurian dam is very interesting and his determination, by flow nets, of the seepage and uplift condition shows advanced thinking. The condition of this dam is very instructive especially as to the necessity of rigid field inspection during construction. It probably reflects the result of that major engineering mistake of divided responsibility, being designed perhaps by competent engineers who did not have control over construction.

Mr. Leliavsky's artful treatment of the writer's paper deserves revelation of its true significance. As a history and textbook on engineering it would qualify as covering the subject from Galileo to just prior to the writer's paper. According to him everything in the paper is wrong, basically erroneous, full of flaws, not clearly defined, neither standard nor rational, dependent upon unorthodox stratagems, imaginative, confusing, uninformed, etc. To prove his points he has subpoenaed as witnesses the great men of the past back to Galileo and, for some reason, has even invoked the German language. Yet, after all this colorful phrasing Mr. Leliavsky, with very minor exceptions, reaches exactly the same conclusions as those upon which the writer relies except those as to the nature of the uplift surface and the interpretation of his experiments.

For example, in Fig. 19 and Eqs. 15, 16, and 18, he arrives at precisely the same conclusions (except in different symbols and words) as does the writer from Fig. 5 and subsequent analysis, stating that:

> "* * * the effective weight of concrete above those elementary columns, and hence those areas, in such intimate contact with an impervious foundation as to exclude water pressure [column A, Fig. 5, or Fig. 19(*c*)], is the full dry weight of concrete plus pore water, or the saturated weight of concrete, whereas the effective weight above areas not in contact [column B, Fig. 5, or Fig. 19(*b*)] is only the buoyant weight of concrete."

However, by using different words, symbols, and drawings he would have the reader believe that he has proved something different. He also neglects to carry his analysis one step further to show that the column in Fig. 19(*c*) becomes buoyant as it does in Fig. 19(*b*) if the surface *xy* were a rough one passing through the lowest pore. In addition, he fails to admit, as shown by the writer, that no water-exclusive contact (which would have to be on glass, rubber, or metal) envisioned by column A, Fig. 5, or by Fig. 19(*c*), can possibly exist in natural foundations, which are always on porous material, and also that the condition in Fig. 19(*b*) is therefore the only applicable condition. Eq. 19, although interesting, is only of mathematical interest for it is based on a theoretical assumption that cannot exist in nature. Likewise most of the discussion is concerned with mathematical elaboration of no bearing upon the points of controversy and only tending to make a very simple problem seem complicated and confused.

Mr. Leliavsky deserves credit, however, for pointing out one fact not always clear in the writer's treatment of buoyancy. Although the writer is correct in stating that the effective weight is the buoyant weight of the concrete, yet (as Mr. Leliavsky points out) the volume of material above any datum to be made buoyant is not the gross volume but the volume reduced by voids or the net displacement. This is correctly stated in Eq. 6, but frequent references to "full buoyancy" must be interpreted as full buoyancy of the solid material of the concrete, not of its gross volume. Thus, 1 cu ft of concrete with a dry weight of 152.5 lb and 12% voids would have a buoyant weight, not of $152.5 - 62.5 = 90$ lb, but of $152.5 - 0.88 \times 62.5 = 97.5$ lb.

If Mr. Leliavsky were to concede the existence of, and the right to consider, a surface, no matter how rough, incorporating a full projected area of pores as in Fig. 34(*d*), he would thus exactly agree with the writer. This is the only argument between the two; since the writer has hereinbefore fully explained his viewpoint and since Mr. Leliavsky depends on his experimental determination of this area, it is next in order to examine these experiments.

The Leliavsky tests[30] involved a very clever principle (new to this problem) of graphically solving an equation containing two unknowns, z and n_w, by creating several simultaneous equations (for graphical solution) through the variation of the load N, Fig. 20. This principle is well described by reference to Mr. Leliavsky's paragraph preceding Eq. 22 and by reference to Fig. 20, except that previous experimenters omitted the load N. Herein lies the principal reason (together with the central hole) why he was able to obtain much more consistent results than previous experimenters.

These tests were conducted, very scientifically with utmost attention to every detail to secure a very high degree of accuracy. As such, they are precisely what they purport to be—namely, a vast improvement in technique over previous tests, almost if not quite to the degree of possible perfection as compared with Föppl, Woodward, Rudeloff, and Panzerbieter, but nevertheless predicated upon the same basic assumption that pore pressure at mn, Fig. 20, causes the fracture. Under these circumstances and with a feeling of admiration for the thorough experimental technique used by Mr. Leliavsky, it is with

[30] "Experiments on Effective Uplift Area in Gravity Dams," by S. Leliavsky Bey, in "Uplift Pressure in and Beneath Dams: A Symposium," *Transactions*, ASCE, Vol. 112, 1947, p. 485, Fig. 27.

the utmost regret that the writer finds it unavoidable to differ with the fundamental conception upon which the tests of all the aforementioned gentlemen were based and which was inherited by Mr. Leliavsky from his predecessors.

Briefly, it is claimed very clearly in his original paper[30] that pore pressure at section mn breaks the specimen and that, considered as a free body, above the surface of fracture mn, at the moment of failure, the only force acting in an upward direction is the uplift force of the water pressure in the pores of the section mn, determined by the average hydrostatic pressure p_m applied to an unknown proportion n_w of the geometrical area of the plane section A_s,—this value of n_w to be determined by the experiment. This uplift force is opposed by the tension z in the specimen and by the sum of all downward forces F, principally mechanical, or (see Eq. 22) $n_w A_s p_m = F + z$.

Mr. Leliavsky states (necessarily inadvertently), in the paragraph preceding Eq. 22, that:

> "* * * $n_w A_s p_m$ is also the uplift force transmitted by the percolating water, through pore action, to the porous material in the top of the specimen [seepage force above kk] * * *."

If this uplift force acts both at mn and above kk, Eq. 22 would necessarily read $2 n_w A_s p_m = F + z$. Mr. Leliavsky is not quite as clear and concise on this point in his discussion as he is in the original paper;[30] but it must be assumed that he still considers that the pore pressure at section mn breaks the specimen —otherwise there would be no relation between breaking strength and n_w at section mn which he claims to determine by the experimental strength.

Obviously this uplift force cannot act both at mn and above kk. The writer holds that there is absolutely no uplift force at section mn, and the only uplift force is the seepage force above kk. This need not have any relation to the "superficial effective porosity" n_w at section mn. It is related only to the mechanical strength of the specimen at mn. Considering the specimen above mn as a free body there can be no force resulting from pore pressure except as, and where, a differential pressure occurs—that is, above kk. A poreless bar of metal, or any other tension tie between kk and LL of the same tensile strength as the concrete specimen at mn, could be substituted for the specimen. If an imaginary material of the same cross section as the specimen, without pores but of the same tensile strength, could be substituted for the specimen (just far enough below line kk to permit entrance of sufficient water for upward seepage), the specimen would fail, nevertheless, at section mn through solid poreless material if that proved to be the weakest point in the specimen in mechanical tension. Thus the pressure in the pores at mn has nothing to do with the failure, and the failure data have no relation to (and cannot therefore be used to determine) the porosity characteristics of the surface of failure mn.

The following has partly been stated by the writer in the main paper just prior to Eq. 8: Pore pressure is uniform (in a vertical direction) between LL and kk (actually not quite true near kk, but sufficiently so for this purpose). Neither stresses nor strains can exist as the result of uniform pore pressure in any specimen; the condition between LL and kk is substantially as if the specimen between these points were merely submerged in the water. Any break in

the specimen between sections LL and kk, therefore, is the result of tension applied above section kk, as if applied by a mechanical tension testing machine attached to the specimen above kk. The total amount of tension thus applied is the measure of the strength of the specimen when subjected to high hydrostatic pressure seeping radially toward the central drainage hole. It has no other significance and is not necessarily indicative of the strength of the concrete specimen in air. The only force of local origin tending to cause elongation of the specimen between sections LL and kk is that resulting from Poisson's ratio for the radially acting compressive force of the external water pressure on the cylinder. (The latter was plainly implied to be of no importance although Mr. Leliavsky magnifies this into a major point.) It exists only in the event of, and to the extent of, any difference between external water pressure and water pressure in the central hole.

Mr. Leliavsky apparently agrees with these basic principles (as witness the fifth paragraph after Eq. 25, and also the pressure diagram to the left of Fig. 20 which shows uniform pressure below kk), but he continues to maintain that his tests determine the "superficial effective porosity" n_w at section mn. The position that n_w, as determined by seepage forces above kk (Eq. 22), is equal to n_w at section mn is necessarily predicated upon assumption. The loads $F + z$ can measure only the forces that cause failure. The true equation (preferable to Eq. 8) should be

$$\text{Upward seepage force above section kk} = F + z \ldots\ldots\ldots (41)$$

The "upward seepage force" is not a function of n_w at any one point, having no relation to mn and little, if any, to kk. It is difficult to visualize the exact physical significance of n_w as determined by these tests. It would appear to be a composite of the conditions existing from kk to the top of the specimen. It might imply the average effective proportion of area over which seepage forces act from kk upward but would not accurately reveal even that value because of inherent faults in apparatus and interpretation. For example, the flow lines necessarily bend sharply around the corner at k and the distribution of flow lines tends to crowd to the outer surface at first; and then, as the seepage progresses upward, the distribution tends to become uniform over the entire cross section, whereas, the assumption by Mr. Leliavsky,[30] in evaluating this distribution, is that it decreases radially on a logarithmic curve from the applied pressure at the outside to the reduced pressure at the central hole. This is true at mn, but nowhere is it true at or above kk, and this is further evidence that the rupture is considered by Mr. Leliavsky to be the result of pore pressure at mn. If the central tubular metal lining of the hole were to extend well below kk, it would seem to result in nearly uniform distribution at kk, equal to applied pressure, before the water begins to seep upward. Thus, the aforementioned sources of error might be nearly eliminated, permitting uniform distribution to be assumed regardless of the initial difference between outside and central pressures.

It is the writer's opinion that seepage forces also act over 100% of the area, as in the case of uplift forces, and as discussed with a reference to Figs. 3 and 4, but the aforementioned sources of error would be sufficient to obscure this result.

Another very important observation must not be overlooked; perhaps it is the most important of all. The tests are shown to measure seepage forces and these are horizontal (or nearly so) in a dam (Fig. 23(*a*)), whereas uplift forces in a dam are normal to seepage forces and would not be revealed by these tests.

For all the foregoing reasons the writer claims that the Leliavsky tests are incorrect in principle and do not disprove or weaken the 100% theory in any respect.

Mr. Leliavsky does concede one, and only one, "acceptable conclusion" (that the pressure against a dam is exerted progressively along the route of seepage); but he claims this to be an old story. This is quite possibly true but it was new to the writer. It has been promulgated by the writer since 1936 when this paper was prepared.

Mr. Davison, in Fig. 22, predicated his analysis on the basis of a hydraulic grade line dropping more steeply than a straight line to tailwater and thus concave upward. This is in very radical disagreement with Professor McNown's Fig. 23(*a*). The latter is believed to be more scientifically supported and the correct flow net for a triangular dam. The difference with respect to Fig. 23(*a*) would not be great when considering a dam with a normal top width. Fig. 23(*b*) is also believed to be a better picture of seepage through a rectangular dam than Fig. 22(*b*).

The writer does not believe that the compaction of sand, for normal loading, materially influences the permeability coefficient, K, as claimed; it certainly would not affect rock or concrete permeability. He is indebted to Professor McNown for correct seepage patterns in triangular and rectangular dams (Fig. 23), as well as for clearing up a number of other theoretical viewpoints, and eliminating Fig. 9(*c*) as incorrect for a triangular dam.

Mr. Boughton's forceful discussion would make the writer wince if it were not at fault. If he will refer to Fig. 9(*b*) and additional discussion by the writer in this closure he will note that full static pressure is not claimed to exist at the exposed face. The pressure reduces to zero at the face, which is not inconsistent with the fact that the face appears dry—nor is it inconsistent with the fact that the height of the column to be made buoyant by internal pressure also reduces to zero along with the pressure. Furthermore, only pressure, in the horizontal construction joints, is needed to support the writer's theory. As to the weight of water in the pores, adding instead of subtracting from the weight, he is referred to column B, Fig. 5 and related analysis[54a] following Eq. 4. This is also confirmed by Mr. Leliavsky's Fig. 19(*b*) and Eq. 15.

There is no reversal point as claimed, and there could not be, except by applying a vacuum to the face. It requires internal pressure all the way to the face to keep seepage moving and this pressure is commonly assumed to reduce along a straight line as in Fig. 9(*b*), at least in the construction joints.

Mr. Boughton misquotes the writer (by partial omission regarding internal pressure) as saying that "* * * pressure seldom exists 'except near the upstream face' * * *." The correct quotation is that "* * * theoretical pressure [meaning full straight line pressure as in Fig. 9(*b*)] probably seldom exists, except near the upstream surface * * *".

Mr. Boughton is referred to Professor McNown's Fig. 23(*a*) which shows that seepage lines are theoretically horizontal in a triangular dam instead of

being "quite steep" as he claims. Moreover, contrary to his statement, the porosity has nothing to do with maintaining "such a line of saturation." The flow net for any homogeneous material in an earth or concrete dam of any cross section that obeys Boyle's law (and all granular materials substantially do) is identical for an identical shape of section no matter how porous it may be. All that the porosity does is to determine the quantity of seepage; it does not fix the steepness of the saturation line, or any other geometrical characteristics of the flow net.

The writer has nowhere made the claim contained in Mr. Boughton's last paragraph except as to the word "illogical." He does not maintain that existing design practice is necessarily unsafe but that it is illogical. He has listed mitigating circumstances to explain why dams thus designed have proved safe, and more could be added; but there is no reason why engineers should persist in one error just because there are other errors that may compensate. The right policy is to correct all errors, not just those with respect to uplift.

In reference to Mr. McHenry's discussion, the writer cannot visualize any relation between either loose concrete cylinders, or chairs piled one on top of one another, and the problems under discussion.

Mr. Hotes admits an uplift area of substantially 100% but claims exactly 100% to be physically impossible. He raises the old argument that 100% cannot be true because some area of contact—no matter how small—must remain to support the load, as quoted by the writer on the first page of his paper, and the refutation of which forms the one and only basic point of this entire paper. Obviously the writer's efforts have been wasted if he cannot get this one idea across to one of scientific mind. Practically, it does not matter whether the uplift acts over 100%, 99%, 97%, or even 91%, but, as a matter of principle, it does. Mr Hotes is again referred to Fig. 35 and the related discussion.

In a volume made up of cylinders as thus depicted, the uplift area is exactly 100% and yet the contact areas are large. The student needs to exert no stretch of the imagination to be convinced of the strength of this mass in tension, compression, or shear. No theory of microcontacts requiring tensile stresses of from 36,000 lb per sq in. to 400,000 lb per sq in. has to be invented to explain the large uplift area. The mass does not need to float on water, as indicated by Mr. Hill.

The writer is indebted to Messrs. Carlson and Davis for pointing out the small size of pores as compared with common conception concerning them. The experiments made at the University of California, in Berkeley, are interesting indeed. They would seem to indicate that in very dense, perfect concrete the hydration process absorbs initial seepage, and after a few years the pressure becomes zero a few feet from the entrance. It is puzzling, however, to note that in specimen 1, even after seven years, when the pressure reads zero from distances of 2 ft to 8 ft, there is still a steep gradient from the entrance to point 1.5 ft. The anomaly is that this gradient cannot exist without flow.

Unpublished experiments on seepage through concrete specimens are said to have been made at Zanesville, Ohio, during the construction of several dams in the Muskingum Valley. With a dry discharge surface these specimens

refused to pass any water, but, when kept wet or submerged at the outlet end, free seepage resulted. These results apparently are related to the Berkeley tests. If capillary pressure resists or prevents seepage, then its effect might respond to rain or dry weather and also differentiate the part below tailwater from that above tailwater. This is a subject upon which the writer has no opinion; and it is not within the scope of this paper. It would make an excellent subject for field study. The apparent impervious character of these specimens, of course, has no bearing on the problem of seepage through joints and foundation contact.

Regarding the last paragraphs of the Carlson-Davis discussion referring to drainage, the reader is again referred to Fig. 36 and related discussion showing that drainage is not necessarily effective.

If the writer understands the Morrison discussion correctly uplift is considered to be equivalent to any other foundation reaction and is not a source of instability. This is true, of course, of Mr. Morrison's basement floor slab, provided that the downward load exceeds the upward load. It matters not whether the load of the building is supported entirely by concrete-to-earth compressive load or partly by uplift. However, this cannot be true of a dam where horizontal loading is involved. The shearing or sliding resistance is not a function of the total reaction including uplift as indicated by Mr. Morrison, but is a function only of the mechanical pressure, grain to grain, of concrete upon the foundation material. Uplift certainly reduces this pressure, and therefore the resistance to horizontal movement. The theory presented is a very dangerous one.

Nothing in the writer's theory, stated or remotely implied, justifies the weight of criticism in the first paragraph of Mr. Hill's discussion. The writer's theory certainly does not require any weight to be supported entirely by water, and yet Mr. Hill, in his sixth and seventh paragraphs, permits load to be carried by water in lieu of particle-to-particle contact of concrete, a very dangerous and incorrect theory and a direct reversal of the attitude expressed in his first paragraph.

Pore pressure definitely reduces the factor of safety against overturning as well as sliding, and Mr. Hill, not the writer, is the only one to deny it. Internal buoyancy is very real; a wet sponge sinks because the individual fibers are heavier than water—a very simple principle.

In the second paragraph of his discussion Mr. Spielman makes an interpretation directly contrary to the writer's statements. Also, he joins other discussers in criticizing the words "internal buoyancy." In no place does the criticism seem to be applicable to the principle, but only to the words adopted to describe it. The writer considers the words to be accurately descriptive. It merely means that the buoyant force is applied in voids and not entirely on the underside of the submerged body as is so often considered.

Mr. Spielman states:

> "If the body were porous, with interconnected pores, it would not displace its gross volume but only the volume of impervious matter, assuming, of course, that sufficient time is allowed for the full penetration of the fluid into the pores."

The foregoing is a perfect definition of "internal buoyancy" and shows that Mr. Spielman agrees exactly with the writer on this subject, except as to terminology; and he agrees exactly with Eq. 6 which is one of the most fundamental points of the entire paper. However, when Mr. Spielman declares, after Eq. 39*b*, "Now, because the voids are filled, the effective area of uplift is $(1 - n)A$, which is something less than 100%" he makes an unwarranted deduction. What he should state, to convey his true meaning (and also that of his statements following Eq. 40) is that, if the submerged object is to be considered as a whole using its gross volume and ignoring its porosity of 12% and ignoring the uplift exerted in the pores, then the uplift should be applied to only 88% of the bottom area to make it applicable to net displacement and thus compensate for ignored porosity. This is a very good reason why the writer uses the term "internal buoyancy"; it is applicable only to net solids and requires no such principle of compensating errors.

Professor Raes is also confused by the term "internal buoyancy," although he seems to confirm the phenomenon. Assume that the porous object in Fig. 35(*a*) replaces the solid object in Fig. 1. Then a vertical element as ab will intersect the pores. The buoyancy of the object as represented by the sum of many elements such as ab will no longer reflect the behavior of continuous elements, such as a to b, but of several short elements with intervening pores between and with total over-all length of a to b including pores. Each of these pores will be under pressure corresponding to its depth below the water surface and each part of the element ab will be individually buoyant because the pore pressure on the bottom will be greater than on the top. The buoyancy of the object as a whole then becomes the sum of the buoyancy of all the short vertical cylindrical elements. No one has suggested a better name than "internal buoyancy" to describe this situation. It is accurately and perfectly descriptive of what takes place. All the buoyant force is not applied at the bottom, but internally—hence the term. The usual conception of buoyancy is that all the uplift force must be applied at the bottom, which is correct for a poreless solid or a ship's hull, etc; but in the case of a porous submerged object it is obviously applied throughout the mass, if the pores are large enough, and if enough time elapses for the pressure to penetrate the pores. These conditions, of course, are assumed to exist.

Admittedly, internal buoyancy is the same as buoyancy of the gross volume with the bottom area multiplied by $(1 - n)$; but in establishing a principle it is not enough to define something that is merely equivalent.

Professor Raes seems to object principally on the grounds that the surface treated by the writer is not the surface of rupture. That this thesis is not concerned with the surface of separation or rupture has already been amply demonstrated. The weight of that part of the body above the rough surface, in Fig. 34(*d*) or Fig. 35(*c*), is reduced by buoyancy, and, unless the specific application requires and anticipates failure (which it seldom, if ever, does in uplift problems), then this surface is immaterial. Moreover, it seems to have been shown by the McHenry tests that failure, even in shear, is influenced by uplift over 100% of the area, although not along the surface conceived by the writer.

Some discussions would lead one to believe that modern dams are practically impervious even at horizontal construction joints. That is debatable, but it is

not the question under debate in this paper. If true, why is uplift still considered? Allowance for uplift is still a part of design technique in even the most modern engineering agencies, usually assumed to act over two thirds, or like proportion, of the area. The theme of this paper is that it acts over 100% of the area. Any discussion of other matters is extraneous to the purpose of this paper.

The writer was once employed to place a new face, about 2 ft thick, on a gravity dam the surface of which was badly disintegrated. The new slab was certain to be much less pervious than the old concrete; and, although this new work was anchored back into the dam with sufficient bars to resist the entire static pressure, it was nevertheless desirable to drain known seepage through this new slab to prevent the building up of serious pressure behind it. The dam faced south and in daytime, with the surface moisture subject to evaporation, the surface of the dam seemed dry except in a few isolated places. To locate the areas of slow seepage it was necessary to inspect the face in the morning shortly after daylight. Many more seeping areas were then visible, including construction joints.

This situation is confirmed by Professor Ballester in connection with the driving of a tunnel into Dique San Roque in Córdoba, Argentina. An apparently dry face as the result of evaporation was found to be deceptive and the interior "weeped" when exposed. This also agrees with the writer's quotation from the paper by the late A. J. Wiley, M. ASCE.[9]

Undoubtedly very great improvement has been made in dam building technique such as in low water-cement ratio, aggregate mixtures requiring less cement, low heat cement, cooling of aggregates and water, use of ice, circulating cooling pipes, thorough cleaning by air and water of construction joints between pours, and grouting of contact areas; but the answer is that, even with this technique, bad contacts occur and consideration of uplift has not been abandoned. Therefore, the theory of uplift should be on a correct and realistic basis even if some other error compensates. It should also be placed on a realistic basis.

By common practice lifts are about 5 ft thick. One lift is allowed to cool from three days to five days before being covered with the next lift. In one of the most modernly constructed dams the writer observed frequent hair cracks on the surfaces of each lift before receiving the next, indicating shrinkage. The new lift is thus placed on top of a surface already partly cured, cooled, and shrunk. The new lift will tend to exert a shear stress at contacts that will require strong restraint to prevent creeping. The writer was told that actual lifting and opening of hair cracks around the edges of a pour at the construction joints had been observed at this dam due to warping of the new pour by more rapid cooling of the surface, much as a board curls up at the edges lying flat in the sun. The movement was very small and would substantially close again; but nevertheless it indicates the possibilities of imperfect contact. The only logical course to follow by those who claim that uplift can now be prevented is for them to show the courage of their convictions and omit it from consideration in dam design. The writer knows of no one who has this degree of courage

[9] "Past Experience with High Dams," by A. J. Wiley, in "High Dams: A Symposium," *Transactions*, ASCE, Vol. 95, 1931, p. 134.

and the writer is not one of those ready to concede omissions of uplift. The entire theory herein presented is necessary to support the existence of 100% uplift at construction joints even if impervious concrete between joints were to be conceded.

The writer makes no pretense of having solved all the problems of dams. His central theme is merely that uplift acts over 100% of the area. To prove this point it is not necessary to attack all the other numerous illusive problems. He has therefore adopted for illustration the most common orthodox assumption—that the magnitude of internal pressure in pores or joints varies from reservoir pressure at the back to zero or tailwater pressure at the face—well aware that this assumption is subject to modification. He also assumed a homogeneous dam without cracks or joints where all water seeks the face to escape (for example, the flow net in Fig. 23(*a*)). The writer is well aware that such a perfect condition does not exist in practice but the assumption aids and simplifies theoretical treatment. Adoption of these and other orthodox assumptions is not to be interpreted as endorsement but merely the avoidance of controversy not pertinent to the central theme of the paper and not within its proper scope. (However, these assumptions seem to have increased rather than avoided controversy.) For the same reason it does not seem within the proper scope to reply to all points brought out in discussions which are irrelevant to this central theme even though the writer was himself guilty of some such departure.

There are many puzzling problems regarding dams. When water was admitted against the Norris Dam of the Tennessee Valley Authority (TVA), it deflected upstream against the water pressure instead of downstream. Was this due to deformation of the reservoir bottom by the added water load, or to cooling of the back of the dam by water while the face remained exposed to the sun? Let the reader answer. Also, Mr. Riegel has shown[44] from observations on TVA dams that delayed central cooling apparently partly relieves central loading and transfers it toward heel and toe. This appears to be one of the most significant facts to come to light in recent years and may go far toward explaining the apparent stability of sections considered deficient. The Carlson-Davis discussion might imply that very great capillary forces exist in a dam capable of exerting a major influence in its behavior.

Likewise there is reason to believe that temperature, cooling, shrinkage, elastic deformation of dam and foundation, plus the foregoing enumerated effects, may even outweigh in importance the forces, stresses, and strains from water load, uplift, and foundation reactions which are the only ones considered in orthodox design practice.

The fact is that very little is known, scientifically, about the design of gravity dams and the engineer's confidence in practical rule-of-thumb methods is based largely upon the good behavior of dams thus designed. Little by little, one problem at a time may be fully studied and better understood until the art may become a science. The writer can only hope that he has contributed to a better understanding of one of the perplexing problems.

[44] "Structural Features of Hydraulic Structures," by Ross M. Riegel in "Design Developments—Structures of the Tennessee Valley Authority: A Symposium," *Transactions*, ASCE, Vol. 111, 1946, p. 1159.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EXPANSION JOINT PRACTICE IN HIGHWAY CONSTRUCTION

Discussion

BY WILLIAM A. CONWELL

WILLIAM A. CONWELL,[22] M. ASCE.—The profession is indebted to the author for compiling and presenting field test data on this highly important subject. Likewise the Public Roads Administration and the participating state highway departments are to be commended for the well conducted and successful series of tests which have done so much to remove the behavior of concrete pavements from the realm of speculation to the field of knowledge.

The writer would like to enter a minor objection to the use of the term "incompressible" in connection with foreign material that enters joints and prevents the closures required to take up the pavement expansion. Since all known materials are compressible to some degree, the substitution of "material of low compressibility" is recommended for "incompressible material."

The tests show that, after the initial expansion and contraction, the so-called contraction joints behave as both expansion and contraction joints. Thus, the expression "contraction joints" is a misnomer. Despite considerable thought, the writer has been unable to produce a term that would be more descriptive of the action. This point may be one of challenge to the profession, particularly to those members engaged in highway construction who can, undoubtedly, develop an excellent terminology from their picturesque vocabulary.

NOTE.—This paper by A. A. Anderson was published in September, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1948, by Frank H. Gardner; and March, 1949, by K. B. Woods, and J. K. Knoerle.

[22] Gen. Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

THE RELATION OF URBAN EXPRESSWAYS TO MASS TRANSPORTATION FACILITIES

Discussion

BY ELMER F. STEIGELMAN, AND HAROLD M. LEWIS

ELMER F. STEIGELMAN,[2] ASSOC. M. ASCE.—Anyone who drives a car will agree with Mr. Bradfield when he states (see "Introduction"), "* * * it is obvious to everyone that traffic conditions in metropolitan areas have become intolerable and some remedy must be found." It might be added that the more intolerable the condition the more drastic the remedy would have to be. The paper presents the problem precisely, and is timely.

Having hotels, stores, theaters, and offices concentrated in a downtown area of a city is an advantage of modern living and decentralization of these facilities is not to be desired. Rather one must accept the fact that there is a centralized downtown area, that people will come to this area in their own cars instead of using public transportation, and that this traffic, together with other traffic, creates an intolerable traffic condition. The writer would like to contribute the following discussion on this condition as it affects the downtown area only:

1. The author suggests that relief for mass transportation could be obtained by removing it from the surface of the street to either subways or elevated structures. However, this procedure is quite expensive and a better one would be to prohibit entirely the use of the downtown area streets by private cars. Traffic in a downtown area is composed of private cars, trucks, and public conveyances (street cars and buses). Of these, trucks are necessary to bring goods to stores during business hours; public conveyances are necessary for people without cars. Thus, the obvious group to eliminate, and one which would relieve the situation considerably, would be the private car.

2. To compensate for the prohibition of private cars and to obviate the necessity of constructing expensive circumferential expressways around the business district, public garages should be erected at the fringe of the district in sufficient number and size to accommodate the cars that would normally park on the streets in the business district. These garages should be placed

NOTE.—This paper by Curtis G. Bradfield was published in December, 1948, *Proceedings*.
[2] Cons. Engr., Berkeley, Calif.

on the perimeter of the district so that the center of the district would be within walking distance.

If the city would prohibit use of the downtown streets by private cars the incentive for private construction and ownership of the garages would be provided. However, a competent planning organization should determine the location and number of the garages for maximum efficiency. If private financing and construction of the garages could be guaranteed, cities would be relieved of additional strain on their finances. Certainly, the cost of the garages would not add up to the cost of subways and circumferential expressways.

3. If the business district is so large that walking from the perimeter garages would not be convenient, an integrated, circulating, low fare transportation system within the downtown area should be provided. To be attractive to the public these units should be spaced not more than 3 min apart and the fare should be approximately 5¢. There would be no transfer privileges to or from this system. Thus, a person would park his car in a perimeter garage and if his destination were not within walking distance, he would use the circulating transportation system, or a taxi, to reach it.

A public garage system has an advantage over the subway program in that it could be financed privately whereas the subway would probably require bond issues. Time required to secure relief would be shorter for a garage system than for a subway or elevated railway program. By eliminating private cars from the downtown area more safety for pedestrians would be obtained and accidents to vehicles might be reduced.

HAROLD M. LEWIS,[3] M. ASCE.—Although expressways were first developed as approaches to cities through suburban areas, it is now recognized that they should extend into urban areas. Mr. Bradfield accepts this, but states (under the heading, "Relief for Intercity Traffic") that they should be used only for "intercity and cross-the-city traffic" and that "local traffic," which should be reduced in quantity by greater provision and use of mass transportation, can be handled on ordinary city streets with the aid of improved regulation (see "Relief for Local Traffic").

The writer agrees that expressways are justified only in locations where they will be of use to through traffic, but believes that the inner sections of such routes should be so designed that they can also take care of intracity traffic which is moving either to or from the downtown areas or from one side of the city to another. In a small city such intracity traffic may travel only half a mile or more along the expressway, whereas in a large city it will travel on the expressway for at least 2 or 3 miles. Thus, the express route can be used to its maximum efficiency.

Mention should be made, also, of the opportunity for still greater use of expressways through their joint use by private cars and mass transit vehicles in the form of buses operated over the main roadways or of rail facilities located in a part of the right of way reserved for that purpose. Such a treatment of a considerable part of the proposed expressway system in Los Angeles, Calif., was

[3] Cons. Engr. and City Planner, New York, N. Y.

proposed at a "transportation clinic" held in that city in December, 1945;[4] and the Hollywood Parkway was designed to accommodate buses, with special turnouts at which they can pick up and discharge passengers.

The writer agrees that the use of private automobiles in central areas should be discouraged, but would include three additional steps to the provision of "adequate mass transportation" as part of the municipal action necessary to implement such a program. These are: The elimination of the free use of downtown streets for transient parking, the substitution of a limited curb side space for short time parking controlled by parking meters, and supplementary off-street parking facilities. The last of these may be provided in retail centers of small municipalities as free municipal parking lots; in other cases they should be operated on a fee basis. In very large cities some form of municipal subsidy may be required to bring the fees within the range of what the driver is willing to pay.

Although some planners have suggested that the modern city should be crisscrossed by a network of expressways, the writer agrees with the author that such schemes are visionary and impractical and that most of the arterial networks of cities in the United States will continue to be surface level streets. The expressway system within urban areas will not often require more than one loop, or circumferential, route except in large metropolitan areas where both an inner and an outer loop may be desirable. If planned far enough in advance most of the outer route can be of limited access type, constructed at grade with depression or elevation only at connecting thoroughfares.

In many cities the expressway system need be only a single route in the direction of prevailing traffic, or two intersecting routes where traffic is heavy in two directions. In each case it should pass through the city along the edge of the downtown area.[5] In an extreme case an additional link may have to cross the central area. For example, in the New York (N. Y.) region the Mid-Manhattan Expressway shown on the New York City master plan is needed to provide for heavy traffic between the New Jersey and the Long Island, New York, sections of the region.

As cities increase in size, the percentage of people entering downtown areas by private car decreases—primarily because mass transportation has been provided. Experience has shown that when cities continue to grow without comprehensive planning both types of facilities become unsatisfactory because of overcrowding and delays, or both. Good mass transportation has the advantages of cheapness, efficiency of street use in the case of surface transit, and speed in the case of subways. The private automobile has the advantages of comfort and convenience. The solution of the congestion problem in the downtown areas is not simply to ban the private car in such areas and to attempt to shift the entire load to mass transit lines. There must be a balanced transportation system in which both private cars and mass transportation have a place.

The stagnation of street movement in central areas has resulted from three main factors:

[4] "City of Los Angeles—Recommended Program for Improvement of Transportation and Traffic Facilities in the Metropolitan Area," by De Leuw, Cather and Co., Harold M. Lewis, and Joe R. Ong, Los Angeles, Calif., December, 1945.

[5] "Express Highway Planning in Metropolitan Areas," by Joseph Barnett, *Transactions*, ASCE, Vol. 112, 1947, pp. 643–644, Figs. 2 and 3.

(1) City planning that failed to coordinate the design of the street system with control over the bulk of buildings fronting on the streets, and that laid out new arterial routes without giving thought to where the cars they bring in will go or park in the downtown area;

(2) Laxity of municipal authorities in permitting downtown streets to be used as free storage space for long time parkers, either through failure to impose restrictions or through inability or unwillingness to enforce restrictions when imposed; and

(3) Failure of both public and private transit companies to modernize and keep up to date their mass transit facilities.

If large cities will stop providing free parking space for transients and substitute fee parking, economic forces will help to bring about a solution. Such fee parking can be provided through meters for short time parking, where curb side space can be set aside for such use, and through off-street parking in lots or garages for long time parking. Most of the workers previously driving into the area and parking all day will then have to shift entirely to mass transit lines or drive only to convenient outlying transfer points. The few who can afford to pay for downtown off-street parking can continue to drive in. This arrangement should leave room on the streets for the salesmen, visitors, executives, and shoppers who will, by either necessity or preference, continue to come by private automobile.

City planners are more often accused of urging decentralization and being willing to let overgrown cities decay than of viewing decentralization with alarm. The decentralization of residences in metropolitan areas has been a steady trend for several decades and when it involves a shift to planned areas it is beneficial. On the other hand, centralization of many types of business activities has distinct advantages, and cities cannot afford to let their established business centers become choked and blighted by inaccessibility. At the same time cities should plan for some recentralization of business in subcenters rather than permit overbuilding in a single center.

In attempting to prove that mass transportation will keep patrons coming to downtown stores because (under the heading, "Relief for Mass Transportation") "cities with the most centralization have the most extensive mass transportation systems," the author may be putting the cart before the horse. Would it not be more logical to say that cities which have developed mass transportation systems have thereby made possible great centralization? This result has certainly been true in New York City, which extended its rapid transit system into unbuilt areas, thereby attracting a vast resident population to those areas and greatly increasing the congestion in the downtown business areas where all the rapid transit lines focused.

The author's proposed cycle of downtown traffic restrictions, increased bus patronage, subway extension, additional outlying bus transportation, and so on, seems to offer hope of a more efficient transportation system, but its effectiveness will depend on how these facilities are planned. If the new transit lines continue to focus on the old downtown center, there is danger of establishing a vicious cycle of new transit lines, bordering residential development, demand for additional parallel transit lines, additional bordering development, and so on—all leading to greater concentration and congestion in the downtown area.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MINERAL LAND SURVEYS

Discussion

BY CHARLES P. EAGER, AND ROBERT LENON

CHARLES P. EAGER.[15]—The presentation of this comprehensive paper is opportune in view of the current activity in mineral land surveying. The scope of the engineering technique (which, of course, is of paramount importance in a paper of this nature) apparently covers the subject. Only where the author touches upon matters pertaining to United States mining laws and to the Bureau of Land Management practice do a few questionable points arise.

An adequate discovery of mineral within the boundaries of any mining claim, whether it be lode or placer, is prerequisite to valid location. Therefore, the specification under "Types of Mineral Claims: Placer Claim" that

> "* * * the discovery [upon a placer] must be not just a discovery of mineral, as with the lode claim, but it must be a discovery of commercial deposits of placer material * * *"

appears to be, at least, an unauthentic differentiation—based, no doubt, on a singular court decision.

The statement in the same section of the paper, as to: "* * * an over-all length of 2 miles or 3 miles being not uncommon for a 160-acre gulch placer" probably applies to the earlier official mineral surveys. It is a fact that the rule enunciated in the mining regulations fixing a limitation on the length of a placer claim will not be applied where the mineral deposits are confined within a narrow strip of land in the bed and on the banks of a small stream in a canyon flanked by abrupt walls or rocky slopes on each side, containing no mineral, agricultural, or timber value.[16] However, the matter of whether a placer location conforms sufficiently to requirements is a question of fact for determination by the Bureau of Land Management in the light of the showing made in each particular case, and the locator should bear in mind that it is not the current policy of the federal government to sanction entries or locations that cut the public domain into long narrow strips or grossly irregular and fantastically shaped tracts.[17]

NOTE.—This paper by Arthur J. McNair was published in November, 1948, *Proceedings*.

[15] Cadastral Engr.-Mineral, Bureau of Land Management, Region II, U. S. Dept. of the Interior, Glendale, Calif.

[16] "Land Decisions," U. S. Govt. Printing Office, Washington, D. C., Vol. 53, 1933, p. 431.

[17] *Ibid.*, Vol. 37 1909, p. 250.

The Act of Congress of May 10, 1872, the regulation[6] which, as Professor McNair states under the heading, "Location Surveys for Lode Claims," still prevails in 1948, apparently did not specify that a lode location—other than a lode within a placer[18]—could not be less than 25 ft on each side of the lode line. Section 2320 of the Revised Statutes states: "* * * nor shall any claim be limited by any mining regulation to less than twenty-five feet on each side of the middle of the vein at the surface."

The smallest legal subdivision of the public surveys provided for by the mining laws is a tract of 10 acres, normally in square form; and it is very improbable that, in the location and entry of placer mining claims, rectangular "tracts as small as 2½ acres" (as stated under the heading, "Location Surveys for Placer Claims") would be recognized and treated as legal subdivisions.[19]

Rules and regulations of the Bureau of Land Management very definitely do not require that the boundaries of a placer located by legal subdivisions be staked because, to stake a placer in this manner, a survey obviously would be required.[20]

In preparing a location certificate for a placer that is to be officially surveyed for patent, it also is very important to include the name of the claim.[21] Under the mining laws of certain states (which are supplemental to those of the United States), this feature is mandatory (Public Resources Code of California, Section 2301).[22] Also, the possessory right to a mining claim is taxable in some states —as, for example, in California. (Section 3617 of the California Political Code defines "right to the possession of land" as real estate (see People versus Scherer, 30 California 645).)

In perusing the section on "Patent Surveys," especially the opening and closing paragraphs, and the statement following the subtitle, "Field Notes," an uninformed reader quite probably would infer that an application for an official mineral survey is tantamount to a patent application. However, these are two separate and distinct procedures. The application for a survey is merely a preliminary action, addressed to the Public Survey Office, for the purpose of obtaining an official description of the claim. After the approval of the survey by the Public Survey Office, there is no limited period (if ever) during which the mineral claimant may make the actual application to the District Land Office for a patent to the property.

Mineral land is not "the only type of public domain patentable in the United States," as stated under "Conclusion." Disposals are currently being made through the Bureau of Land Management in practically all classifications permitted under the public land laws.

---

[6] Act of Congress, May 10, 1872, Section 10; Revised Statutes of the United States, Sections 2329, 2330, and 2331; 30 United States Code, Sections 35 and 36.

[18] Revised Statutes of the United States, Section 2333, U. S. Govt. Printing Office, Washington, D. C., 2d Ed., 1878.

[19] "Land Decisions," U. S. Govt. Printing Office, Washington, D. C., Vol. 34, 1906, p. 260.

[20] *Circular No. 1278*, Bureau of Land Management, U. S. Govt. Printing Office, Washington, D. C.

[21] "Code of Federal Regulations," U. S. Govt. Printing Office, Washington, D. C., 1940, Pt. 185.14, title 43.

[22] *Circular No. 127*, California Div. of Mines, State Printing Office, Sacramento, Calif., 1944.

ROBERT LENON.[23]—In commenting upon Professor McNair's excellent paper, it might be well to note, that, although the writer has held a commission as mineral surveyor since 1932 except during World War II, his experience has been limited to Arizona and consequently this discussion will be based on such local experience.

Under the heading, "Location Surveys for Lode Claims," Professor McNair mentions that "* * * since 1920 the majority of location surveys has been made and the descriptions have been written by engineers." Such is not the case in Arizona; the writer, for example, has made very few such surveys in the course of almost 20 years of practice, and it is quite unusual to find location notices on record with bearings any more specific than "westerly, southeasterly, etc."

Professor McNair mentions the necessity for "bending" claims to keep all the vein within limits ("Types of Mineral Claims: Lode Claim"). A fairly common case is that in which a bend in the outcrop is caused solely by the topography, the vein itself being as straight as possible. For example, consider the case of a rather flat-lying vein striking across a flat, and up a hill. Where the slopes of the hill and of the vein intersect (on the outcrop), a definite deflection will be apparent on the horizontal projection, increasing as the hill steepens or as the dip flattens.

It is not the present policy of the Land Office to require side-center monuments to be set, in Arizona, or to be mentioned in the field notes, although end-center monuments (or their counterparts at each end of the lode line when the lode does not cut the end line exactly at its center) must be set.

Although he has not seen the law in writing, it is the writer's understanding that, in Arizona, a mill site may not be preempted or patented contiguous to a lode claim.

Although the length of the lode line may not exceed 1,500 ft, the side lines may be longer—in fact, up to a length of almost 1,530 ft. The latter is possible in the case of a keystone-shaped claim with a 1,500-ft lode line, with one end line 600 ft long and the other, say, 10 ft long. However, triangular claims are not allowable, no doubt because of the ensuing extralateral rights which require differentiation between side lines and end lines not possible with a figure of this shape.

In the case of lode claims, no limit is placed, by Arizona laws, on the number of claims an individual may hold, although each mining district may enact such limitations. With placer claims, however, each locator may hold but one 20-acre claim, whereas an association may hold 20 acres for each individual within the organization. Although the laws on this matter are rather vague and incomplete, they have been interpreted to mean that an eight-man group, for example, can hold 160 acres of placer ground either as eight 20-acre claims or as one 160-acre claim.

Professor McNair mentions the inclusion of a level rod in his survey outfit, but does not state how it might be useful during the actual prosecution of the survey aside from its obvious use as a stadia rod for a rough check of taped or triangulated distances. The writer's party carries a rod quite frequently as an

[23] Cons. Mining Engr., Mineral Surveyor, General Land Office, Patagonia, Ariz.

adjunct to the work of claim surveying for patents. It is useful for "spotting" less important workings, buildings, and roads. In the frequent cases where a wide canyon is spanned by a long tape it is quite handy to note the position of the center of the gulch by stadia with sufficient accuracy for the purpose, and such improvements as roads and pipe lines may be readily located by traversing by stadia from convenient points (not necessarily corners) along any of the traverse lines being taped. As in other such work, it is well to tie in both ends of such a traverse rather than to leave it with no position check on the far end.

Professor McNair does not mention that mineral surveyors are precluded, by Land Office ruling, from acting, directly or indirectly, as attorneys for the claimants seeking patents. Many claimants believe that the mineral surveyor is the person to aid them in filling out their requests for survey orders and other subsequent required paper work. Although, in some respects, he is the logical person to consult in these matters he is not normally fitted by training (or temperament!) for this phase of the procedure.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

INTEGRATING THE EQUATION OF NONUNIFORM FLOW

Discussion

BY PHILIP Z. KIRPICH, AND PAUL V. HODGES

PHILLIP Z. KIRPICH,[9] ASSOC. M. ASCE.—As stated by the author, Boris A. Bakhmeteff, Hon. M. ASCE, first devised an integration method, applicable to all shapes of channel, for the determination of nonuniform or varied flow profiles. As further expressed by Mr. Von Seggern (see "Introduction"):

> "The Bakhmeteff method, however, evaluates only a part of the integral, thus making it necessary to apply a factor or a series of factors to account for the remainder."

Although these factors are by no means arbitrary, their determination requires further computation and plotting. The author has successfully eliminated much of this labor by:

(*a*) Providing a chart for the exponent n; and
(*b*) Introducing an exponential factor m and developing tables for two new functions, designated Y and Z, in which m and n are parameters.

The same purpose can be achieved by retaining Professor Bakhmeteff's procedure and developing charts for the exponent n and the critical slope σ.[10] Although the factors given in these charts are for rectangular and trapezoidal shapes only, values can be determined for circular, parabolic, or other shapes. From Fig. 10 it is seen[11] that n is a function of y/b only and can be read more directly than in Fig. 2. The critical slope σ is also easily obtained from Fig. 10 except that, in this case, it is necessary to divide by the factor $\left(\frac{1.486}{n}\right)^2 b^{\frac{1}{3}}$, in which n is the Manning coefficient. With the exponent n and σ known, the integration is readily performed using Professor Bakhmeteff's varied flow function tables.[12]

NOTE.—This paper by M. E. Von Seggern was published in January, 1949, *Proceedings*.

[9] Hydr. Engr., Knappen Tippetts Abbett Eng. Co., New York, N. Y.

[10] "Dimensionless Constants for Hydraulic Elements of Open-Channel Cross-Sections," by Phillip Z. Kirpich, *Civil Engineering*, October, 1948, p. 47.

[11] *Ibid.*, p. 48, Fig. 4.

[12] "Hydraulics of Open Channels," by Boris A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, pp. 308-311.

As applied to Example 1 of the paper, first compute $K_M = \frac{1.486}{n} = \frac{1.486}{0.025} = 59.4$; then $K^2_M\, b^{\frac{1}{3}} = 59.4^2 \times 25^{\frac{1}{3}} = 10{,}300$. Values of n and σ, selected from Fig. 10, are as follows:

	y	y/b	n	$\sigma/K^2_M\, b^{\frac{1}{3}}$	σ	β	$1-\beta$
Station 1....	8	0.32	3.7	58	0.0060	0.07	0.93
Station 2....	6.73	0.27	3.65	62	0.0056	0.07	0.93

A quantity β as defined by Professor Bakhmeteff equals $\frac{S_o}{\sigma}$; and $\eta = \frac{y}{y_o}$.

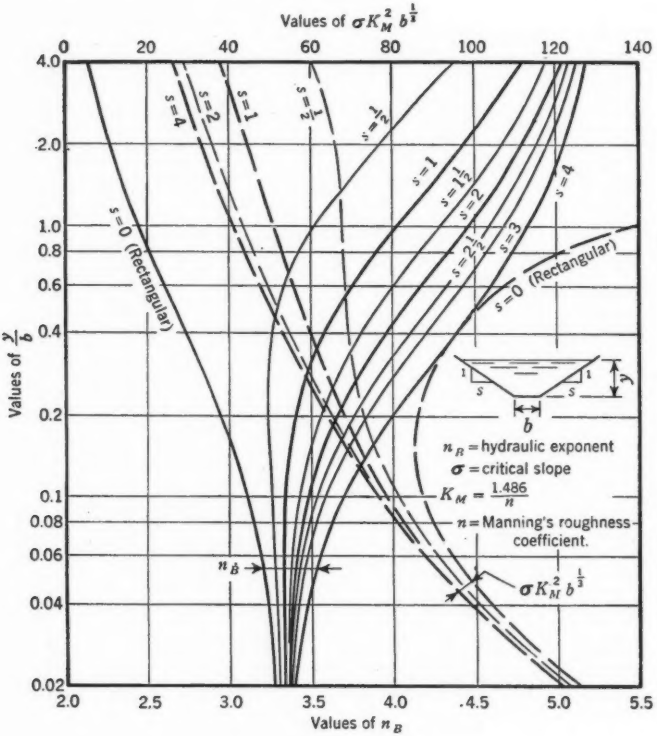


FIG. 10

Then a quantity $B(\eta)$ is obtained from Professor Bakhmeteff's varied flow function tables using an average value of the exponent n as a parameter, and $\pi = \eta - (1-\beta)\,B$. With these definitions and with an average value of n of 3.7 (taken to the nearest tenth), the computation is completed as follows:

	η	$B(\eta)$	$(1-\beta)B$	π	$\Delta\pi$
Station 1.....	1.19	0.303	0.28	0.91	1.46
Station 2.....	1.001	1.668	1.55	−0.55	

—which checks the author's result within 2%. The slight discrepancy is partly caused by the different n-value used—3.7 instead of 3.6.

In cases where alternative procedures are available, the engineer should choose the method with which he is familiar, unless some other method possesses marked advantages. Mr. Von Seggern has, in fact, developed a procedure having the advantage of reducing the labor required in computing varied flow profiles. However, the improvements in Professor Bakhmeteff's original procedure described in this discussion tend to negate this advantage—that is, if the factors in Fig. 10 are used in conjunction with Professor Bakhmeteff's varied flow function tables, the advantage claimed by the author's method is largely lost. At the same time it is unnecessary, using the alternative suggestion presented herewith, to become familiar with two additional functions (designated Y and Z) and to acquire tables of these functions.

PAUL V. HODGES,[13] M. ASCE.—The integration of the equation of nonuniform flow by Mr. Von Seggern is a valuable accomplishment toward a more definite knowledge of different flow problems. Eqs. 11 and 13, together with Figs. 1 and 2 and the tables of Y-functions and Z-functions, make possible the easy solution of discharge problems in channels of uniform cross sections with sustaining slopes. The solution of the typical examples adds greatly to the value of the paper and makes it a very useful reference. Because of the curvature of the lines in Figs. 1 and 2, the values of the hydraulic exponents, m and n, tend to increase with an increase in depth y.

In Example 2, the author used the average values of 3.4 for m, and 3.6 for n in computing the total distance of the surface curve. To indicate the relative effect of changes in m and n, the writer recomputed Example 2 by individual reaches, using the values of m and n as determined for each reach, as shown in Table 3.

TABLE 3.—RESULTS OF COMPUTATIONS FOR EXAMPLE 2, APPLYING EQ. 13

y	m	n	U	L	ΣL
4.05 6.30	3.3	3.6	2.667 1.715	383	0 383
6.3 8.5	3.4	3.65	1.715 1.27	2,391	383 2,774
8.5 10.38	3.55	3.7	1.27 1.04	12,316	2,774 15,090
10.38 10.59	3.55	3.7	1.04 1.02	5,948	15,090 21,038
10.59 10.79	3.55	3.7	1.02 1.001	27,652	21,038 48,690

The difference in total distance of the surface curve as computed by the two integration methods amounts to only 2.32%.

The computation of the surface curve by the ordinary step method requires relatively small steps to obtain close agreement with the integration method. In Example 2, step computations for each 0.2 ft or 0.3 ft for the flatter part of the surface curve would be required to find the distances that would agree

[13] Office, Corps of Engrs., Denver, Colo.

approximately with the integration method, and consequently such detail computations would be long and would require considerable time.

A comparison of the results obtained by the integration method and by step methods with 0.5-ft steps and 1-ft steps is given in Table 4.

TABLE 4.—COMPARISON OF INTEGRATION AND STEP METHODS FOR EXAMPLE 2

Method	DEPTHS OF FLOW y (IN FEET) FOR FIVE VALUES OF L (IN FEET)				
	15,000	21,000	31,000	48,690	49,820
Integration Method, with:					
Five reaches	10.38	10.59	10.71	10.79	....
One reach	....	....	....	....	10.79
Step Method; Depths Increased by Steps of:					
0.5 ft	10.44	10.65	10.80	....	....
1 ft	10.54	10.80	....	....	....

The application of the integration method to natural streams would be very difficult because of the varying cross section and slopes usually encountered. In their present form, Eqs. 11 and 13, could not be applied to channels with horizontal bottom grade. The integration of the equation for nonuniform flow and the computations of the Y-functions and the Z-functions are important advances in the development of the science of hydraulics.

Correction for *Transactions:* In January, 1949, *Proceedings,* on page 120, insert "$-\frac{1}{n}$" before the integral in Eq. 23.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CITIES CANNOT LIVE WITHOUT TRUCKS

Discussion

BY HAROLD M. LEWIS

HAROLD M. LEWIS,[5] M. ASCE.—The timeliness of this paper lies in the fact that it stresses the importance of truck traffic to urban areas, and the importance of so handling such traffic that it may move expeditiously without hampering the other uses of city streets. Any prosperous city must have healthy industrial and business areas, and these require intensive truck services. As the author states, this requisite calls for not only sound street planning, but also the design of buildings and industrial areas for the motor age, with facilities for off-street loading and unloading. Although, to some extent, such facilities can be required under municipal ordinances, adequate provision will in part remain dependent on the architects who design the structures, the engineers and city planners who lay out the street system, and the executives of business and industrial concerns whose cooperation is essential.

Truck Versus Passenger Car Traffic.—There is a wide variation in the percentage of trucks to total vehicles on highways in the United States—not only in regard to the highway, but in regard to the time of day, day of the week, and season of the year. On arterial routes the number of trucks will ordinarily comprise from 5% to 25% of the total vehicles; on some of the bridges in New York City this value reaches 35%. On the interstate crossings operated by the Port of New York Authority trucks average approximately 20% of the total traffic.

In the 1920's, when the modern parkway systems were being developed, stress was laid on the need of express routes for passenger cars. It was assumed that trucks, being slower vehicles, could get along on ordinary streets where they would find plenty of room when some of the passenger cars had been diverted to parkways. The parkway program has encouraged the use of more passenger cars but has not solved the movement of trucks, many of which are now capable of traveling at speeds comparable to those of passenger cars. Looking forward in 1927 to the need for better routes for trucks, the Engineering Division of the Regional Plan of New York (N. Y.) and Its Environs stated that, in addition to the restriction of certain routes to passenger vehicles, it was probable that:

NOTE.—This paper by Hoy Stevens was published in December, 1948, *Proceedings*.

[5] Cons. Engr. and City Planner, New York, N. Y.

> "* * * there will eventually have to be highways which are intended for trucks only, and which passenger vehicles would either naturally avoid or from which they would be excluded."[6]

Limited access highways available for all types of traffic provide equal facilities for trucks and passenger cars. The writer believes that most of the expressways to be built in the future will be of this type. However, they should be supplemented by parkways through suburban sections of metropolitan areas and along scenic routes. Perhaps, some day, certain highways will be reserved for trucks during the weekdays.

Compulsory Off-Street Loading.—The provision of off-street loading and unloading facilities not only should be required in new buildings exceeding a specified minimum floor area designed or used for such purposes—manufacturing, offices, hotels, etc.—as require service by large numbers of trucks, but also should be made retroactive for buildings of the same types. Such retroactive provisions were recommended in 1942 by the Regional Plan Association, Incorporated, in its study of traffic and parking in New York City.[7] This recommendation was based on observation of the extent to which such facilities had been voluntarily installed, and suggested a five-year period for compliance. An essential part of any such regulation would be provision for a variance in cases where undue hardship would result. The Regional Plan Association recommended the following provision to cover such cases:

> "Where, due to an unusual shape of the lot or structural features of the building, the Board of Standards and Appeals shall determine after public notice and hearing that these provisions cannot reasonably be complied with by the owner, that Board may approve such alternative provisions for truck loading and unloading as it deems suitable and adequate to prevent the creation of serious traffic congestion by such loading and unloading operations."

Such retroactive provisions, to be effective within a period of from five years to ten years, have also been suggested by Nathan Cherniack,[8] M. ASCE. On main highways outside central business districts municipalities, through their police powers, may require substantial setbacks for industrial buildings as well as the provision of adequate loading and unloading spaces which, in most cases, can then be provided in the open spaces on the lot.

The writer agrees with the author that there is need for better designed industrial districts served by streets on one level rather than for elaborate systems of two-level streets. Nevertheless, in some of the older intensively developed districts it may be necessary to provide two levels, as has been done on Wacker Drive and part of Michigan Avenue in Chicago, Ill., and on the west side of Manhattan in New York City, with an elevated highway for passenger vehicles.

Formulas for Off-Street Loading and Unloading Areas.—Any formula for an off-street loading and unloading area incorporated in a municipal ordinance

[6] "Highway Traffic," Vol. III in "Regional Survey of New York and Its Environs," New York, N. Y., 1927, p. 99.

[7] "Traffic and Parking Study," Regional Plan Assn., Inc., New York, N. Y., December, 1942, pp. 43 and 138.

[8] "A Statement of the Parking Problem," by Nathan Cherniak, *American Planning and Civic Annual*, 1946, p. 129.

must set the minimum requirement for certain types of buildings. The writer questions the feasibility of the author's suggestion (see "Amount of Off-Street Loading Area") that such requirements "* * * should be based on the volume and tonnage of freight to be moved into and out of a building." These factors will vary with the type of occupancy. Any buildings that will have exceptionally heavy truck service should be provided with more than the minimum legal requirements. If the city supplements its off-street loading requirements with regulations of street use to prevent the abuse of curb side space, this result should be obtained.

In the Regional Plan Association study of 1942,[7] it was shown that in many cases structures within this exceptional classification had voluntarily provided much more than the minimum off-street space which would be required under the regulations proposed. For example, a large department store in New York City had eighty-five off-street loading berths in comparison with the minimum of twenty five which would be required for a building of the same size "used for manufacture, storage, or goods display, for a department store, or for a hospital." Again, the underground truck berths in the Rockefeller Center group of office buildings in New York City had forty loading berths as against the twenty two that would be required for a similar floor area "used for office purposes or for a hotel supplying full facilities for transients."

The sliding scale used by the Regional Plan Association for computing the number of truck berths, referred to by the author ("Amount of Off-Street Loading Area"), was based on probability tables used by the American Telephone and Telegraph Company in determining the number of trunk lines necessary under different loads to avoid keeping a customer waiting too long to make his connection. It was developed under the assumption that 1 truck out of 4 might have to wait for one eighth of the average loading time.[9]

The minimum sizes of buildings that should come under such compulsory loading space restrictions will vary with the size and character of the community. In New York City the minimum size for buildings under the present regulations is 25,000 sq ft of gross floor area. The Regional Plan of New York and Its Environs recommended that such requirements apply to loft, department store, or office buildings having 8,000 or more square feet, when located in "close suburban or intermediate areas."[10] In several ordinances for small communities a minimum size of building is specified but the amount of off-street space to be required is to be determined by the zoning board of appeals.

Smaller buildings in established centers must continue to be served by curb side space, but cities should forbid the use of sidewalks for the storage and classification of motor truck freight. This regulation would require owners to provide adequate ground floor storage and elevator service to upper floors for such freight.

Supplementary Types of Regulations.—Among the other types of regulations which cities may enforce to relieve street congestion by trucks are:

[9] "Traffic and Parking Study," Regional Plan Assn., Inc., New York, N. Y., December, 1942, p. 43.

[10] "The Building of the City," Vol. II in "Regional Plan of New York and Its Environs," New York, N. Y., 1931, p. 176.

1. Keep large trucks handling less-than-truck-load shipments out of congested business areas by the provision of union, intercity truck terminals where their freight will be sorted and reloaded on smaller vehicles for delivery. The same procedure in reverse would apply to pickups. Two such terminals in the New York metropolitan area were under construction by the Port of New York Authority in 1949.

2. Limit the hours for pickup or delivery of truck-load lots in congested business areas. This would not necessarily require night service, but might ban any service during the heaviest rush-hour periods.

3. Designate certain streets in central business areas for the routing of trucks desiring to pass through such areas without pickup or delivery.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

NOMOGRAPHIC ANALYSIS OF RECTANGULAR SECTIONS OF REINFORCED CONCRETE

Discussion

BY MAURICE BARRON, AND M. HIRSCHTHAL

MAURICE BARRON,[4] M. ASCE.—One of the important features of the nomograms in this paper is the fact that they contain only nondimensional quantities which permit their use, without adjustment, in either the metric or the English system of units. Because of this nondimensional property, the nomograms are universal in that they can be used for any specification, any combination of stresses, any arbitrary concrete strength, and any embedment of reinforcement.

For several years, the writer has used nomograms[5] by Henri Dumontier virtually identical to these for the review of arch sections; but for the design of the sections other methods are usually preferred which do not require tables, charts, or diagrams.

Fig. 1 is of particular interest, in that it shows the minimum amount of steel reinforcement (tension steel plus compression steel) required for the design of a concrete section subjected to combined bending and direct stress. This nomogram, as well as the others, is based on the straight-line theory, and currently the plastic theory[6] indicates that a stronger structure results when part of the reinforcement intended as compression steel is used as additional tension reinforcement—because of the redistribution of stress resulting from plastic yield.

An interesting and enlightening problem is presented as follows:

The cross section is 12 in. by 24 in., subjected to a moment of 35.6 kip-ft and a direct thrust of 45 kips; depth $d = 21$ in. The specifications permit: $f_s = 18{,}000$ lb per sq in.; $f_c = 1{,}000$ lb per sq in.; and $n = 10$. By Eqs. 1, $B = 0.1573$ and $C = 0.0002$. Using Fig. 1, with these values and $k = 0.357$, the solution is not apparent. From a direct design method the following solution results—$e = \frac{M}{N} = \frac{35.6 \times 12}{45} = 9.5$ in.; $a = \frac{1}{2}t - 3 = 9.0$ in.;

NOTE.—This paper by W. P. Linton was published in January, 1949, *Proceedings*.

[4] Chf., Structural Design Div., Clarke, Rapuano & Holleran, New York, N. Y.

[5] "La Technique des Travaux," by Henri Dumontier, Société des Pieux Franki, Liége, Belgium, February, 1928, pp. 135–146.

[6] "Plastic Theory of Reinforced Concrete Design," by Charles S. Whitney, *Transactions*, ASCE, Vol. 107, 1942, p. 251.

$e' = e + a = 18.5$ in.; and $M_s = N e' = 45 \times 18.5 = 832$ kip-in. For a balanced section with the full allowable stresses—$k = 0.357$; $j = 0.881$; $K = 157.3$; $M_B = K b d^2 = 832$ kip-in.; and $A_s = \frac{M_s}{f_s j d} - \frac{N}{f_s} = \frac{832}{18 \times 0.881 \times 21} - \frac{45}{18} = 2.50 - 2.50 = 0$ sq in.

Therefore, this section, subjected to the designated stresses, requires no tension reinforcement and no compression reinforcement. It should be noted that the thrust is considerably outside the middle third of the section and that, for an uncracked section design, tension greater than 200 lb per sq in. is indicated. The paradox becomes more complicated if the embedment in the foregoing problem is 2 in. instead of 3 in. With an increased depth d the required tension steel changes from zero to 0.2 sq in. If the embedment is increased to 4 in., no tension steel is required, but compression reinforcement is necessary. This appears to be another defect in the straight-line theory, and similar results will be obtained frequently for sections that have values of C equal to or approaching zero. In Eq. 1*c*, the quantity in brackets approaches zero when e'/d approaches the value of $1 - \frac{d'}{d}$.

Since the compression bar in Fig. 4 replaces an equal area of concrete, p' should be multiplied by $(n - 1)$ instead of by n in the derivation. This point is usually ignored for practical simplification in the derivation of the equations and construction of the nomograms. It is a simple procedure to use the nomograms as shown and to apply a correction to the area of compression steel. Mr. Dumontier introduced two nomograms similar to Fig. 1 which the writer considers more convenient in arrangement and scale layout. In these nomograms the paradox demonstrated by the foregoing example falls outside the range and the designer cannot be misled. The unusual case requiring special consideration is brought forcefully to the attention of the designer.

The author is to be commended for a very clear presentation and for making these nomograms readily available to the profession.

M. Hirschthal,[7] M. ASCE.—Mr. Linton has certainly performed a valuable service for the designers of reinforced concrete structures who have the problems of designing for bending and direct stress, or who use steel as compressive reinforcement. The labor expended by the author on the preparation of his nomographic charts is undoubtedly enormous. He has saved each individual using them a corresponding amount of time.

It is all the more regrettable, therefore, that he did not take into consideration the fact that both the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete[8] and the American Concrete Institute in the latest design specifications[9] have given consideration to the fact that the modular ratio in the compression side varies from that of the tension side of a concrete member reinforced for compression as well as tension. The regulation

[7] Concrete Engr., D.L. & W.R.R. Hoboken, N. J.

[8] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, ASCE, June, 1940, Pt. 2, p. 45.

[9] *Journal*, A. C. I, June, 1945, p. 582.

adopted by both these bodies is based on the assumption that the value of n (the modular ratio) for compressive reinforcement is double the value used for the tensile reinforcement. This assumption was adopted because of the low stresses to which compressive reinforcement was subjected under the old assumption of a uniform n on both sides of the neutral axis of the section, and was the result of test observations[10] made on large size columns at the University of Illinois, at Urbana.

Under the old provision, compressive reinforcement in a 3,000-lb concrete, the modular ratio (in tension) of which is assumed as 10, would be stressed to about 9,000 lb per sq in. For a beam such as Fig. 4, in which d is 16 and d' is 2 in., the stress in the compressive steel will be $\frac{5.5}{7.5} \times 1{,}000 \times 10$—which is somewhat more than 7,000 lb per sq in., or somewhat less than one half of the stress capacity of the steel. However, if the value of n for the compression side is taken as double that in tension, the compressive reinforcement will be stressed to about 16,000 lb per sq in., taking advantage of the full capacity of the steel in compression.

The writer appreciates the fact that the basic formulas will have to be modified both for the location of the neutral axis and for the resulting percentages of steel reinforcement p and p' in Eqs. 14. However, since $\boldsymbol{B}$ and $\boldsymbol{C}$ are not affected, the variation is not quite as serious as it otherwise would be.

It is to be hoped that the use of 2 n as a modular ratio for reinforcement in compression will be generally adopted since it will result in a marked saving in the reinforcement required in compression.

Such a percentage of saving is certainly worthwhile even at the expense of considerable design calculation labor. This saving will be about 50% as the position of the neutral axis is but slightly affected by this change in the modular ratio.

Corrections for *Transactions:* In January, 1949, *Proceedings*, on page 135, line 6, change "$p\ n$ should be * * *" to "$p'\ n$ should be * * *"; in Eq. 16*b* place a minus sign before "$(1 - k)$" in the numerator of the second quantity; and, in Fig. 4, the depth d' should extend to the center of the bar instead of to the top.

[10] *Bulletin No. 40*, Univ. of Illinois, Urbana, Ill., June 5, 1934, p. 86.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

HIGHWAY BRIDGE FLOORS
A SYMPOSIUM

Discussion

BY LOUIS BALOG

LOUIS BALOG.[31]—In general, in the design of a bridge, it is not necessary to consider the framing effect of the slab alone. Actually, a great variety of efficient structures can be designed by disregarding the framing effect of the slab entirely and providing load distributing diaphragms. In this connection Professor Newmark states (Section 2*b*):

> "Since the slab acts as a very effective diaphragm, it is unnecessary to provide additional diaphragms, except for construction purposes, if the slab can perform its function of distributing loads to the beams at the same time that it provides a roadway for the wheels to roll over. Where it is expedient or desirable to make the slab thin, and therefore flexible, some additional transverse bridging is desirable. In general, however, such bridging is not particularly effective except for loads at or close to the section where the transverse frames are located."

The foregoing statement applies only to small bridges of the type investigated by the author. When properly designed, framed bridges are an advantage because diaphragms assure that the structure will behave as contemplated in the design for its entire life and, if desired, for the entire range of loading, up to capacity loads. The cost of efficient diaphragms is insignificant as compared with the great advantages that result from their use.

In addition to their definite and permanent action, diaphragms facilitate the application of new ideas in floor construction. High quality pre-cast floor units of low weight can be manufactured at an efficient plant with a strength and durability far superior to those attained in the field. Such units can be pre-stressed; they can be provided with special connecting elements; and they can be used as forms for the part of the slab that is cast in place. In long-span work, particularly, the wide range of possibilities for controlling

NOTE.—This Symposium was published in March, 1948, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1948, by Searcy B. Slack; October, 1948, by E. W. Wendell, and H. Tachau; and November, 1948, by Reto Furrer.

[31] Cons. Engr., Binghamton, N. Y.

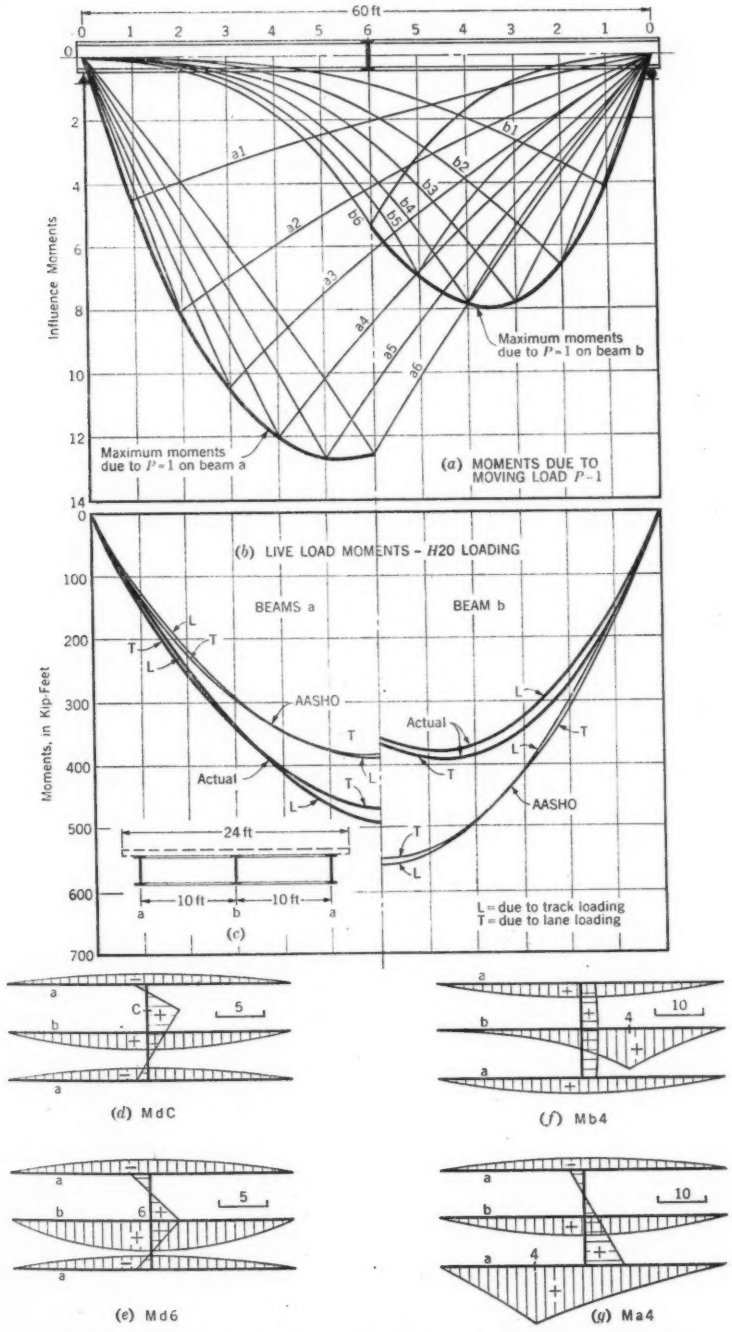


FIG. 42.—EFFECT OF A CENTER DIAPHRAGM ON THE MOMENTS IN A 60-FT SPAN

the weight of the slab is of great significance. All these advantages are possible only by the use of diaphragms.

The writer questions, especially, the part of the foregoing quotation in which Professor Newmark states:

> "* * * such bridging [transverse bridging or diaphragms] is not particularly effective except for loads at or close to the section where the transverse frames are located."

Any rigid transverse connection between longitudinal girders (or trusses) will influence the deflections of the connected girders. An applied load will be distributed to all members of the framework in proportion to the magnitude of the deformation resulting from this load. Fig. 42 will serve to illustrate the writer's point of view as applied to a 60-ft span, the bridge having three girders connected by a center diaphragm. The flooring consists of a two-way slab with bays 10 ft square. The diaphragm causes the outside beams to support the center beam at the point of connection, which means that the center beam acts as a continuous beam on an elastic center support when it is loaded. When an outside beam is loaded, the unloaded beams behave in a similar manner but to a lesser degree. A moving unit load on an unframed beam creates a maximum moment of 15, whereas the introduction of the diaphragm results in a comparable maximum moment of 8 in the center beam (beam b, Fig. 42), the point of maximum moment being shifted toward the quarter point of the span. If the moving unit load is on an outside beam (beam a, Fig. 42) it causes a maximum moment of 12.7 by the introduction of the diaphragm, and the point of maximum moment moves about only one twentieth of the span length from the center.

Evaluation of fourteen influence lines of this framing (four of which, MdC, Md6, Mb4, and Ma4 are shown) for H-20 track and lane loadings (see Fig. 42) demonstrates: (1) That the diaphragm is a very effective load distributor in the entire length of the bridge; and (2) that the maximum values of the moments in the outside beams are 23.2% and 25.8% larger, whereas in the center beam they are 47.9% and 57.4% smaller, than the American Association of State Highway Officials (AASHO) specifies.[18] The maximum stress in the diaphragm is 96% larger than that obtained according to the specification. Load distribution, such as that indicated in Fig. 42, is characteristic for bridges that have been built whether the girders are framed by diaphragm or are framed predominantly by floor slab. The latter is the case—for instance, in the Little Miami River bridge at Xenia, Ohio.[32] Evaluation of the deflection measurements[33] of this 5-beam, two-span continuous bridge, for which H is approximately 5 and b/a, approximately 0.1, reveals that the AASHO specifies 32.6% smaller loads and 38.8% and 50.9% larger loads for the outside and interior beams, respectively, than the actual values. In this case also, the deviations of the specifications from the actual are of the same order as for the bridge with a center diaphragm shown in Fig. 42. The recommendations of

[18] "Standard Specifications for Highway Bridges," AASHO, Washington, D. C., 4th Ed., 1944.

[32] "Vibration Studies of Continuous Span Bridges," by L. E. Vandegrift, *Bulletin No. 119*, Ohio State Univ. Eng. Experiment Station, Columbus, Ohio, July, 1944.

[33] "Load Distribution Over Continuous Deck Type Bridge Floor Systems," by W. S. Hindman and L. E. Vandegrift, *Bulletin No. 133*, Ohio State Univ. Eng. Experiment Station, Columbus, Ohio, May, 1945

Professor Newmark (Section 6*b*, also Eqs. 3 and 4, and Table 2) deviate from 13.4% to 32.7% from the values measured at Xenia. They constitute, therefore, some improvement of the specifications, but they fail to approach reality to a satisfactory degree.

A case in point is the 1,068-ft-long, continuous plate-girder bridge over the Elbe River at Stillhorn-Neuland, Germany (1938). The spans are 193 ft, 341 ft, 341 ft, and 193 ft; the girder spacing is 20.34 ft; the floor beams, 5.3 ft deep, extend 1.5 ft above the girders and their triangular brackets extend to the bottom flanges of the girders. Measurements[34] of the load distribution among the four girders of this bridge disclosed behavior similar to that indicated by Fig. 42. Many other cases of this nature could be cited to support the writer's contention concerning the action of diaphragms. The Xenia measurements indicated that the slab acts as a diaphragm, which knowledge is not new. Comparative calculations show, however, that with a properly connected diaphragm of insignificant cost the same effect can be achieved reliably.

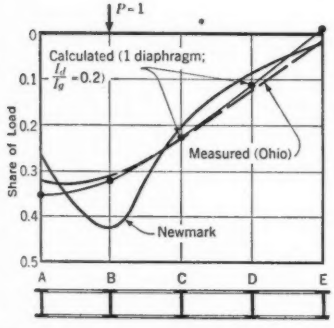


FIG. 43.—DISTRIBUTION OF LOAD $P = 1$ ON BEAM B IN A 5-BEAM BRIDGE

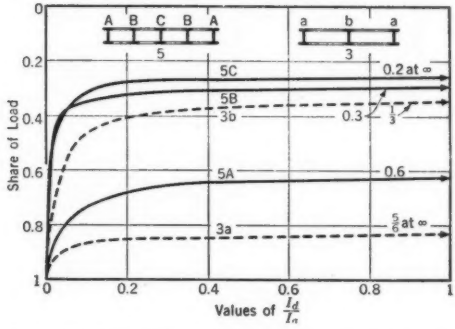


FIG. 44.—SHARE OF LOAD $P = 1$ ACTING ON THE BEAM FOR VARIOUS I_d/I_g RATIOS IN A 5-BEAM AND A 3-BEAM BRIDGE

Fig. 43 affords a comparison between three sets of values for the transverse distribution of a load at midspan of beam B, in a 5-beam bridge: A computed value assuming one diaphragm with $I_d/I_g = 0.2$ (subscript d denotes "diaphragm" and subscript g denotes "girder"); values reported by Professor Newmark (Figs. 14 and 15); and values measured at Xenia. The latter values and the computed effect of the center diaphragm are practically identical, whereas the Newmark values (especially the value of 0.428 P determined for beam B) are considerably different from the value of 0.309 P, as measured. Field measurements on other bridges revealed distribution lines similar in character to those obtained at Xenia, whereas measurements on a model of a bridge with an 125-ft span and five girders framed by four evenly spaced diaphragms, but without the deck slab[35] ($I_d/I_g = 0.21$ and $b/a = 0.129$), produced lines similar in character to those in Fig. 14. The effect of this change in the load-distribution diagram due to the action of four diaphragms as com-

[34] "Brücke über die Süderelbe im Zuge der RAB Hamburg-Hannover-Bremen," by Max Roloff, *Die Bautechnik*, Vol. 17, 1939, pp. 481–488 and 513–517.

[35] "Anleitung für die vereinfachte Trägerrostberechnung," by Fritz Leonhardt, Wilhelm Ernst & Sohn, Berlin, 1940, p. 91.

pared to that distribution produced by a center diaphragm was found to be small on the stresses in the connected girders, but not for the diaphragm stresses which were considerably reduced. It can be stated generally that the stresses in girders remain practically the same regardless of the number of load-distributing diaphragms greater than three, even in spans longer than 300 ft.

Fig. 44 shows the effect of the diaphragm and girder stiffness ratios on the distribution of load placed on one of the beams and carried by that beam. The number of beams are, respectively, five ($b/a = 0.1$) and three ($b/a = 1/6$). This diagram demonstrates that comparatively flexible but properly connected diaphragms are efficient load distributors between closely spaced beams. Furthermore, with an increase in the spacing of the beams the stiffness of the diaphragm gains in importance. In the last sentence of Section 2*b* Professor Newmark states:

> "Such a procedure [plastic theory of limit design] would be applicable and justified only when the slab or other transverse framing remained stiff enough at the higher loads to distribute the loads equally to all the beams."

Fig. 44 shows that this criterion for the safety of the application of limit design principles is impossible because, even by the use of infinitely rigid transverse framing, the loads on all the beams cannot be made equal.

The use of rigid floor beams, however, is advantageous if such beams (diaphragms) are justified structurally. They unite the girders with the lateral systems into an efficient space structure. The deflections of such floor beams are negligibly small as compared with those of the girders they connect, and therefore the load distribution among the girders and the stresses in the floor beams can be determined from the vertical and torsional rigidity of the connected system by comparatively simple calculations. The 3,095-ft-long, 90.55-ft-wide "Arpád" bridge over the Danube River in Budapest, Hungary (1938-1947), is such a structure.[36] It contains three continuous girder units in which 197 ft and 338 ft are the shortest and longest spans, respectively. The four girders are spaced at 23.8 ft, 20.7 ft, and 23.8 ft. Floor beams of the Vierendeel type extend the full depth of the girders. It was found that the positive area of the load-distribution diagram of the outside girders is 24.15% larger than that of the interior girders; the outside girders carry 52.2% of a load uniformly distributed between the interior girders; and they carry 53.05% of the load on the two electric railway tracks located between the two interior girders. The extent to which the AASHO specifications are faulty, in preventing the outside girders from sharing the load placed between the two interior girders, is apparent from these data.

The specifications, and the opinions of Professor Newmark concerning the action of diaphragms, will also prevent the proper design of a system comprising widely spaced girders and widely spaced, load-distributing, rigid floor beams (diaphragms), between which grids consisting of shallow floor beams and stringers are provided. If such a system is well proportioned the economies that are attainable are quite considerable. By 7,800 individual measure-

[36] "Construction du Pont Arpád sur le Danube à Budapest," by Ch. Széchy, *L'Ossature Métallique*, October, 1947, p. 413.

ments [37] on two tranverse sections of seven double-track railway truss bridges, ranging from a 138-ft span to a 620-ft span, it was found that such an insignificant transverse connection as a shallow top strut changed the distribution of the load on one track between two 66-ft-deep trusses 16%.

In his paper, Professor Siess reports the comparative behavior of small scale models of smooth flanged beams without, and with, special provisions for hindering the movement of a mortar slab at the top of these beams. In the "Summary" he cites savings in metal from 8% to 30% in favor of the latter arrangements. In appraising the practical value of this conclusion the behavior of bridges under their design loads cannot be disregarded. Measurements of the deflections of actual bridges having concrete decks resting on the tops of the girders, whether riveted or welded, were found to be from 30% to 65% smaller than those determined theoretically under the assumptions made during the design.[38] Considerable deviations between the calculated and actual deflections and the strains in bridges of this type were observed consistently, irrespective of the age of the structure. What happens under ideal laboratory conditions, therefore, does not indicate the actual behavior of bridges built without special shear connectors.

Measurements [39] on a 557.74-ft-long, two-span, continuous, through, plate-girder, bridge having a concrete deck on five roadway stringers and on two sidewalk stringers, and a lateral system at the bottom of the floor beams, 5 ft deep, disclosed that the strains in the extreme top and bottom fibers of the girders were from 9% to 21% smaller than calculated despite the fact that the loads were large enough to cause 3% remaining deflections. The reduction of the strains was evidently the result of the composite action of the floor, the laterals, and the girders. It is a significant fact that—even in this type of girder—the stress was 21% smaller than that stress which would be used in deciding the relative economy of this structure.

The economic comparisons by Professor Siess give undue advantage to a type of bridge over other types whose actual behavior he did not report. Therefore, in structures of different quality these comparisons depict girders with shear connectors in which the allowed stresses are reached at ordinary working conditions; and they depict others in which the stresses are much lower, permanently. Considering further the behavior of framed structures during times of overload, the comparative quality of bridges that rely entirely on the effectiveness of the connectors is also lower at the higher loads.

Two-way slabs are bonded into transverse elements that are framed rigidly into the main girders. The economy of this type of construction is enhanced by the triple action of the transverse beams which support and bond the slab and frame the girders. If the beam flanges are extended into the slab large rectangular rims or plugs resist sliding. Even if the bond could be destroyed completely, such a slab would not lose "its ability to transfer shear" (see Part C, Section 2). There are several structural arrangements by which

[37] "Über die Verwindungssteifigkeit von zweigleisigen Eisenbahnfachwerkbrücken," by Rudolf Bernhard, *Der Stahlbau*, Vol. 3, pp. 85–92.

[38] "Composite Girder Bridges," by Louis Balog, *Engineering News-Record*, Vol. 140, 1948, p. 64.

[39] "Neubauten von Strassenbrücken," by Arthur Lämmlein, *Die Bautechnik*, Vol. 18, 1940, pp. 25–29.

effective composite action accrues as a by-product and in which additional resistance to slab sliding can be created by simple details. The significant fact is, however, that composite action of all the parts of bridges occurs in actual structures regardless of the conscious effort made by designers to guarantee it. In order to produce bridges of uniform strength and quality and to compare the economy of various types of bridges on a fair basis it should be recognized that comparison of the complete action of one type with the action of other types, without considering the true composite action of their floors, framings, and lateral systems, cannot yield consistent results.

On the basis of their observations, the Symposium authors declare transverse framing to be ineffective, and define a rule for load distribution. In the opinion of the writer, these observations appear too limited in scope to support general conclusions. Load distribution is a function of the geometric and stiffness characteristics of the girders and the framing, and therefore cannot be generalized or fixed by standard specifications. A bridge or a floor system can be laid out so as to be the most advantageous solution of a given problem by regulating the load distribution in the design. Such regulation can be accomplished by varying: (*a*) The geometry of the framing; (*b*) the stiffness ratio of the longitudinal and transverse elements; and (*c*) the stiffness ratio of the girders. Three properly designed intermediate floor beams or cross frames will distribute bridge loads practically as effectively as any larger number of frames. Therefore, floor beams or cross frames required for purposes other than load distribution need not be of the same section as the frames provided specifically for the purpose of load distribution.

The behavior of the actual structure is the integrated result of the composite action of all its parts. Research can help to advance design procedures by segregating, quantitatively, the component effects of the actual behavior of the structures, thus making possible the simple calculation of the true simultaneous action of such effects in any practical combination by which the designer wishes to achieve a desired structural performance.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE LOGICAL SYSTEM OF FREIGHT RATES

Discussion

BY ALEXANDER MARKOWITZ

ALEXANDER MARKOWITZ.[4]—The thesis of this very brief paper is that the existing system of freight rates is illogical and that a new system is required. To the writer it seems that one-half dozen pages is too little space for disposing of this complicated subject. Many important factors have thus been omitted from consideration—the complex nature of the nation's industrial life; the geographic pattern of seacoasts, ports, rivers, and harbors; the number and kinds of transportation services available in different parts of the United States which are dependent on the volume of traffic; the costs of handling such traffic; and the effects of competition. In formulating a new system, the author may properly be criticized for oversimplifying a complex, controversial problem, on a mechanical, formulated "template," which is not so readily adapted to the engineering approach.

Density of traffic, the value of the commodity, and the service on which the ability to pay in freight rate making is based, for example, have a direct effect on the level of freight rates. It is thus that commodities of higher value can be required to bear a fair share of the transportation burden, and that raw materials can move cheaply to points of manufacture; yet, these factors receive no consideration in the paper. That there are distinct differences between types of transportation and between carriers of the same type, and that different costs result from their operations, do not enter into Colonel Hall's proposal for a new system.

Actually, all kinds of business are located everywhere, and there are no industrial reservations in which only certain kinds of manufacture are located to the exclusion of others. In a rational approach to any new system the entire effect of geographical distribution of wealth, raw materials, manufacturing skills, agriculture, mining, and other kinds of industrial activity must be taken carefully into account. To the writer, it seems that the only way that the author's proposal could be put in effect would be to make a few individuals or

NOTE.—This paper by Charles L. Hall was published in November, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1949, by Raphael H. Courland; and February, 1949, by Otto Meyer.

[4] Gen. Traffic Mgr., New York and New Brunswick Auto Express, Highland Park, N. J.

groups of men responsible for rearranging the nation's industry and its commerce to conform to his pre-arranged system, and thereafter administer the freight rates by engineering formula.

The new system proposed would eliminate competition quite effectively; and, if that is the price that the nation must pay to obtain the ultimate in simplicity in rate-making, one problem may seemingly be solved only to create many new ones, more dangerous than the present problems. Finally, Colonel Hall proposes a system of engineered differentials to keep each form of transportation in some allocated place. The fundamental fallacies that some body of men represents the entire transportation industry (or that it should) and that it would be possible to leave the element of competition to regulation by the engineering profession are involved in the author's assumption.

According to the author it should always be possible to transfer goods from one prescribed class to another by unanimous agreement of all carriers. To the present time it has been impossible to gain unanimous consent even among one type of carrier. The Interstate Commerce Commission is having a difficult time establishing a uniform classification of freight even between rail carriers. What would be the elements comprising a basic unit cost of transportation by which the public (presumably through the Interstate Commerce Commission) could fix freight rates as outlined in the paper? Rail carriers may be using a 40-ft boxcar, motor carriers a 30-ft trailer, and steamships their sundry volumes of hold and deck space. How would the author suggest that light and bulky articles be classified uniformly so as to obtain the unanimous consent of all, on some weight and measurement basis?

The author would fix rate schedules so that they would need revision only once every 10 years, as against the present thousand changes or more each day. The engineered differentials would be fixed at 5-year intervals. The proposed basic factor (which is not defined clearly in the paper) would be examined once each year. The dynamic transportation industry, with its numerous facets of of commerce, would be placed on a drawing board; it would be examined with dividers, engineers' scales, slide rules, and other engineering implements, until the plan conformed with the assumed dictates of a fixed formula and then it would be set aside marked "not to be opened" for 1, 5, or 10 years, as the case may be. The author shows scant interest in, or sympathy for, the competition for markets and the rivalry between ports. Presumably one or two ports would be selected to move the traffic, and the remainder would be employed for some other, unspecified, use. This would be the purest kind of "planned economy" which, finding itself at odds with reality, and inadequate to meet the needs of a dynamic commerce, would propose to limit commercial freedom to gain its ends. It would stop the pulse of commerce, destroy all initiative and free enterprise, and would substitute the approach of an engineered problem held in check by rigid timed formulas to produce a preconceived result.

The writer can assemble no words strong enough to condemn the philosophy of this paper. It would "stop the clock" and stifle progress in transportation forever. It would be the opening step in establishing a technocratic state, because the same engineered formulas could be applied with equal impunity to every form of enterprise. It would impose a superplanning board to run things

as the engineers would have them, and would substitute drawing-board planning for real lifeblood thinking and action. The freight rate problem is a difficult one. Possibly it is overcomplex, and no doubt changes could be made to simplify its structure, equalize opportunity, and create better adjustments. This, however, is not a stagnant need, to be regulated as by a valve and turned on and off at will. It is ever present in the living organism of commerce which transportation serves. To treat it as the author proposes would require control by an iron hand motivated by an iron will. Colonel Hall's theories must be rejected in their entirety. They hold no single suggestion of improvement by natural changes and constructive planning. To accomplish what he proposes, it would first be necessary to destroy the forces that make the present situation seem so illogical. To do so would certainly be a high price to pay for logic applied to one activity in an otherwise illogical world. Even in the engineering profession, the place for the drawing board and slide rule is subordinated to dynamic thinking and planning. These devices are servants—not the master—and so it is in transportation.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CADASTRAL SURVEYS AND RESURVEYS

Discussion

BY SCOTT P. STEWART AND CLARENCE S. JARVIS, AND ROGER E. AMIDON

SCOTT P. STEWART[2] AND CLARENCE S. JARVIS,[3] M. ASCE.—This timely paper presents some of the salient facts related to both the history and the scope of public land surveys in the United States, which have proved to be adequate and satisfactory after a test covering some 164 years of national growth and development. As a practical application of the coordinate system (which, in recent years, has made notable headway among street layouts for metropolitan and other urban areas), the subdivision of public lands into townships, sections, and quarter sections unquestionably facilitated the early settlement following these surveys. The system validated and protected the holdings of those courageous frontiersmen who had established homes in the wilderness prior to such surveys.

The types of corners prescribed and authorized by the "Manual of Instructions for the Survey of the Public Lands of the United States" seemed to cover every possible contingency—a charred stake or a quart of charcoal under a mound of earth; a durable stone about 18 in. long, or a timber post 3 ft long and 5 in. square, buried two thirds in the ground; suitable markings on the exposed edges or on the faces of a marker to indicate its relation to the township boundaries or the distances in miles therefrom; and accessories such as mounds of stone, or pits, or inscriptions on trees, to be placed according to prescribed patterns. Further clues as to distances are afforded by items noted in the survey records, such as crossing ledges, ridges, or stream banks, and as to alinement, by blazing or by removing the bark from restricted parts of tree trunks near the survey lines.

Basic Data Assembled During Survey and Reconnaissance.—The importance of these cadastral surveys seemed to escape well-merited recognition until recent years; and even at this time, full appreciation has not been accorded in some quarters, as richly as deserved. The typical United States deputy sur-

NOTE.—This paper by A. C. Horton, Jr., was published in January, 1949, *Proceedings*.

[2] Cons. Engr., Salt Lake City, Utah.

[3] Cons. Engr., Salt Lake City, Utah.

veyor for the General Land Office used some elementary and technical astronomy daily in order to determine solar time, latitude, and true meridians. His field notes revealed some knowledge of geology, botany, and hydrography; otherwise he could not have made adequate detailed descriptions of ledge outcrops, vegetation, or woods through which the survey lines were run, and the springs, brooks, and rivers crossed or located within convenient range of the section boundaries. Even the crude estimates of discharge as recorded during these surveys, together with the dates of such observations, certainly afforded some valuable data for prospective settlers or investors in such regions.

The prevailing stands of native grasses, undergrowth, or shrubs mentioned in the field notes afforded valuable clues to the normal seasonal precipitation depths and distribution. The oft-repeated comment, "Good grass for grazing," together with mention of native wild life including game and also whatever range live stock were encountered or observed, must have wielded a potent influence in the development and utilization of areas suitable for grazing. Likewise, notations giving the precise location of outcroppings, either on the survey lines or adjacent thereto, probably provided the first authentic information concerning unusual mineralized formations, exposed coal strata, oil shales, fissures filled with hydrocarbons such as asphalt and elaterite, and springs or streams covered with iridescent oil slick. Even the estimated or clinometer measured depths of canyons and heights of ridge crests or of beetling cliffs are of practical value for those contemplating journeys or homemaking ventures within the regions to be opened for homesteading, following such surveys.

The Township Plats.—The writers have in their possession a print of "Township No. 27 South, Range No. 16 East, of the Salt Lake Meridian, Utah," issued as a public service by the Bureau of Land Management (under date of June 5, 1947), which was selected almost at random as representative of rugged, broken country, but also of very recent date. The township is located in southeastern Utah, a few miles west of the Green River. According to the plat, there are notable Indian paintings in sections 17, 18, and 19. Isolated improvements such as a pumping plant and watering troughs evidently have to do with stock watering facilities; ranch houses and other improvements depicted on the plat will insure the owner of first consideration if he but files his claim within the prescribed preferential period.

The various witness corners, designated by the initials "W C," are placed on firm footing to witness the actual corner point whenever it falls on inaccessible ground, or within a river bed, or on the steep and insecure slope of a deep ravine.

The system of surveys devised for the public domain so many years ago undertakes to carve out the maximum number of quarter sections that can be defined within a nearly rectangular tract. For example, a township is 6 miles long from north to south. If they are based on a standard parallel of latitude, the south boundaries of each township have the same length, this dimension decreasing progressively for each township toward the north (but with all the accummulated shortage thrown into the west tier of quarter sections); and the nothernmost boundary is further decreased in length as an adjustment to allow for the convergence of meridians. The ridge crests and drainage channels are

connected across the sections for the township plat, thus testing the skill and observation of the United States surveyor.

At costs ranging from about 3¢ per acre to 10¢ per acre, or, say, $9 per lin mile to $27 per lin mile, the public land surveys have assembled and depicted salient basic data of incalculable value to both settlers and government.

Inherent Adventures.—Early survey projects were fraught with the dangers lurking in hostile frontiers and adjacent country. Although these had largely disappeared by the beginning of the twentieth century, it is indisputable that some of the more difficult and rugged terrain still awaited the original surveys.

The writers were associated on the U. S. General Land Office surveys of both the Uncompahgre and the Uinta Indian reservations in eastern Utah, from 1901 to 1904. The first named assignment covered the lofty Book Cliff range with its mammoth outcrops of bituminous coal near Thompsons, and extended northward along Willow and Hill creeks for 50 miles toward Ouray, the western boundary skirting the notorious "Robber's Roost." The "wanted men" of that hideout preferred to pass by unnoticed, to avoid conversation, and to accelerate the forward march of survey parties.

While retracing early survey lines 20 miles westward from the Duchesne Bridge, in June, 1903, for the purpose of locating the initial point for that season's contract surveys, the writers had occasion to question the wisdom of declining the military escort proffered by Captain Mercer, United States Army commandant at the Whiterocks headquarters, some days earlier. A platoon of Indians clothed in gay colors and mounted on their best ponies came charging up the slope and then rode in a circle about the transitman and recorder with threatening gestures and angry shouting. The senior deputy surveyor, keeping his eye at the transit telescope, and apparently oblivious to the imminent danger, merely called, "Get off the line; I can't see through you." The Indians held a powwow, then gradually withdrew toward their camps. Thereafter, the surveyors were under continuous surveillance throughout each day for some weeks. The Indians would lurk in hiding in the vicinity of activities, but occasionally they would peer over a ledge or a ridge or from behind a tree. They demolished the witness mounds of stone, scattering them for some rods from the corner point, but apparently did not recognize the actual corner monuments, mostly buried securely in the ground with notched edges showing above the surface.

Extensive surveys nearer the two-state corner in northeastern Utah, on which the writers were similarly associated, involved the usual meanders for navigable waters along the banks of the Green River in 1906 and included some of the gorges and dam sites now under study for flood control, water conservation, irrigation, and hydroelectric projects. Requisitioned boats and rope ladders facilitated access to some critical points; but the transfer of pack train livestock by swimming from the right bank to the left bank, and of personnel and camp equipment by raft during high water season involved the more colorful adventure. The final loading proved to be an overload; the raft began to submerge in midstream, and thus the saturated equipment added to the overload. With navigation thus made more difficult, the raft was drawn into the rapids instead of landing upstream; the anchor ropes cast to shore and

quickly passed about tree trunks, snapped like threads, and tragedy loomed until the raft struck a submerged boulder squarely, and balanced precariously until all the personnel and equipment had been rescued.

Errors of Closure.—Improved surveying equipment such as long steel tapes instead of the four-rod surveyor's chain with its hundreds of wearing surfaces, also the use of clinometers for measurements on slopes, instead of "plumging" or leveling the chain segments, and employment of engineering students as chainmen helped to reduce the errors of closure about the circuit. Finally, the daily observations on Polaris by the hour-angle method in the gathering twilight out on the survey line (rather than waiting for lantern lighted operations at either culmination or elongation during night hours at the camp site) seemed to make the usual closing limits even in mountainous country more nearly 5 ft or less per section instead of the maximum allowable limit of 2.8 rods per section (46.2 ft per section), and proved to be a morale builder as well.

ROGER E. AMIDON,[4] ASSOC. M. ASCE.—The interesting and educational paper by Mr. Horton is modest in its presentation of the enormous job performed in public land surveys since 1785. The comparative ease of locating, identifying, and establishing land lines in wild areas of the west is immediately apparent to anyone who has encountered similar problems in the undeveloped sections of the eastern United States.

The accurate identification of corners and lines on good aerial photographs permanently records the correct location with respect to land and topographic features. The adaptation of this procedure is certainly a great step forward. The hachured General Land Office plats have been of material aid in assisting field identification. However, the use of contours or form lines instead of hachures, in presenting relief, would facilitate identification in many instances.

During the process of preparing the standard United States quadrangle maps considerable effort is expended locating and identifying land net corners. Usually only a relatively few of the corners are actually identified and located in the field. The remainder are established from General Land Office plats, cultural features such as old fence lines, and information secured from local residents. The inevitable result is that errors occur and lines appear on the final map which are incorrect with respect to topographic and cultural features. The only absolutely dependable corners appearing on the sheets are those shown as "recovered corners." A cooperative arrangement between the Bureau of Land Management and federal mapping agencies whereby land net corners would be recovered or established prior to, or at the same time as, the mapping would result in precise presentation of land lines.

Incorporation of the land net into the geodetic control system throughout the United States would ultimately result in significant economy. Areas will be remapped and surveyed for as long as development takes place and, consequently, a need will always exist for correct geodetic control. The tendency in remapping is toward the adoption of larger scales, with increased detail and precision. This in turn demands a greater intensity of control positions. Township corners tied into the geodetic system with third-order accuracy would

[4] Civ. Engr., Flood Control Surveys, U. S. Forest Service, Berkeley, Calif.

provide control for small-scale mapping. Section corners established with this precision would furnish adequate control for all but the most precise surveys. Bulletins showing the coordinate position of township and section corners with specified order of accuracy would furnish private and public agencies a base from which to extend surveys for practically every purpose.

The accomplishment of the ideals suggested herein would require changes in administration of public land surveys; but the need exists and over-all economy would ultimately result. Possibly some thought and study has been given to this problem by officials of the Bureau of Land Management, or others. If so, the results or recommendations would be of interest to the engineering profession.